

Obtaining the specific heat of hadronic matter from CERN/RHIC experiments

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Abstract The specific heat of hot hadronic matter is related to particle production yields from experiments done at CERN/RHIC. The mass fluctuation of excited hadrons plays an important role. Connections of the specific heat, mean hadronic mass excited and its fluctuation with properties of baryon and electric charge chemical potentials (value, slope and curvature) are also developed. A possible divergence of the specific heat as $1/(T_0 - T)^2$ is discussed. Some connections with net charge fluctuations are noted.

Key words specific heat • particle-yields mass-spectrum

Introduction

The statistical model of very high energy collisions can account for particle production yields from very high energy collisions [2–6]. The same model also contains information regarding the thermodynamic properties of this system of particles. One important thermodynamic property is the specific heat. In this paper, expressions for the specific heat will first be developed. The importance of a study of the specific heat stems from the fact that sudden changes in the specific heat have been used as signals for phase transitions. A classic example of this statement is the lambda transition in liquid helium. The name lambda transition reflects the lambda shape of the specific heat with a very sharp rise followed by a sudden decrease. In the liquid-gas phase transition of nuclear matter at moderate excitation energy or temperature a very similar sharp peak in the specific heat was found in a theoretical model developed in Refs. [7, 8]. This is associated with the increase in the surface energy of the system as the original nucleus breaks into small and smaller clusters with increasing temperature. For the situation discussed here, a rapid rise in the specific heat is associated with large fluctuations in the mass spectrum of the excited particles. Event-by-event studies [17] have also been stressed along with temperature fluctuations [18]. Large values of the specific heat are associated with large energy fluctuations. The compressibility is associated with density fluctuations [16]. Fluctuations associated

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Received: 30 September 2005
Accepted: 11 May 2006

with net electric charge and baryon charge have also been of recent interest [1, 11, 12] as well as p_t fluctuations [9]. An overview of fluctuations and correlations can be found in [10, 15]. Connections with some of the quantities that appear in this paper with baryon and electric charge fluctuations will be mentioned.

The organization of this paper is as follows. First, results of the statistical model are given for particle production yields and for thermodynamic quantities in situations where constraints associated with conservation laws are important such as in heavy-ion collisions. The specific heat is then connected to properties of the particle production yields and conserved charges such as baryon number B and electric charge Z . Limiting cases of the specific heat are discussed which show a connection of the specific heat to the mass fluctuation in the spectrum of excited particles. Connections of the specific heat with the behavior of the chemical potential (its value, slope and curvature) are also developed. The distribution of particles obtained from the detailed analysis of fitting the statistical model to hadronic multiplicities in Pb-Pb collisions at 30 A, 40 A and 80 A GeV data [2] is then used to study the behavior of the specific heat. A parameterization of the behavior of the chemical potential with T from this analysis may indicate a sharp increase in the specific heat.

Statistical model

Statistical and thermodynamic properties

The statistical model of heavy-ion collisions assumes that hadron multiplicities are the result of an established thermal and chemical equilibrium [13, 14] in some interaction volume V at some temperature T . The interaction volume is the freeze-out volume which is the largest volume over which equilibrium is maintained in the evolution of the fireball. The statistical equilibrium is developed from the underlying collisions between particles from the strong force. Here, initial reaction rates are assumed fast enough compared to an expansion rate so that a quasi-equilibrium can be achieved. As the system expands, reaction rates quickly drop because of rapidly decreasing densities and equilibrium is broken at some point in the evolution. The simplest assumption is that all particles freeze-out at the same volume V and temperature T . The particle multiplicity distributions in model are then

$$(1) \quad \langle N_j \rangle = g_j \left(V m_j^3 T / 2\pi^2 \right) \sum_k K_2(k m_j / T) \cdot \exp \left[b_j \mu_B k / T + q_j \mu_Q k / T \right] / k$$

The b_j and q_j are the baryon number and charge of particle j which has spin degeneracy g_j and mass m_j . The μ_B and μ_Q are the baryon and charge chemical potentials. The strangeness chemical potential μ_S will be set equal to 0 and the strangeness suppression factor γ_S will be set equal to 1. The main focus will be on the baryon and electric charge conservation in systems with large B and Z . The energy of particle j is given by

$$(2) \quad \langle E_j \rangle = g_j \left(V m_j^3 T / 2\pi^2 \right) \sum_k \left((3/4) K_3 + (1/4) K_1 \right) \cdot \exp \left(k \left(b_j \mu_B + q_j \mu_Q \right) / T \right) / k$$

The arguments of the Bessel K functions in Eq. (2) are the same as in Eq. (1). The energy equation has the particles rest mass within it. The sum over k in the above equations gives the degeneracy corrections, with the $k = 1$ term the non-degenerate or Maxwell Boltzmann limit. For non-relativistic particles in the non-degenerate limit, the

$$(3) \quad \langle N_j \rangle = g_j (V/\lambda_j^3) \exp((-m_j + b_j \mu_B + q_j \mu_Q)/T)$$

The thermal wavelength λ of particle j is given by $\lambda_j = h/(2\pi m_j T)^{1/2}$. The energy in this limit is simply $\langle E_j \rangle = \langle N_j \rangle (m_j + (3/2)T)$. For zero mass particles: $\langle N_j \rangle = g_j (VT^3/\pi^2) \exp(b_j \mu_B + q_j \mu_Q)/T$, $\langle E_j \rangle = 3T \langle N_j \rangle$.

Features of the chemical potentials μ_B and μ_Q

The two chemical potentials are determined by the constraint conditions on total baryon number B and total charge Q or Z . Namely: $B = \sum_j b_j \langle N_j \rangle$ and $Z = \sum_j q_j \langle N_j \rangle$. Moreover, the derivative of these chemical potentials with respect to T can be obtained from the conditions $\partial B / \partial T = 0$ and $\partial Z / \partial T = 0$. Also, use will later be made of $\partial^2 B / \partial T^2 = 0$ and $\partial^2 Z / \partial T^2 = 0$. As an example, consider a system with the multiplicity of all particles given by the non-degenerate non-relativistic limit of Eq. (3). Then,

$$(4) \quad \begin{aligned} (3B/2) + M_B/T + C_{bb} f_B + C_{bq} f_Q &= 0 \\ (3Z/2) + M_Q/T + C_{bq} f_B + C_{qq} f_Q &= 0 \end{aligned}$$

Here, the various quantities that appear in Eq. (4) are defined by the following equations:

$$(5) \quad \begin{cases} M_B = \sum_j m_j b_j N_j, & M_Q = \sum_j m_j q_j N_j, \\ C_{bb} = \sum_j b_j^2 N_j, & C_{qq} = \sum_j q_j^2 N_j, & C_{bq} = \sum_j b_j q_j N_j \\ f_B = \partial \mu_B / \partial T - \mu_B / T, & f_Q = \partial \mu_Q / \partial T - \mu_Q / T, \end{cases}$$

In the above equations $N_j = \langle N_j \rangle$. The coupled equations for f_B and f_Q give

$$(6) \quad \begin{aligned} f_B &= (-3B/2 + M_B/T) C_{qq} + (3Z/2 - M_Q/T) C_{bq} / C \\ f_Q &= (-3Z/2 + M_Q/T) C_{bb} + (3B/2 - M_B/T) C_{bq} / C \end{aligned}$$

where $C = C_{bb} C_{qq} - C_{bq}^2$. If we neglect antibaryons production and take $b_j = 1$ for all baryons (no composites) then $C_{bb} = B$. Mesons do not contribute to either C_{bb} or C_{bq} . Antiparticles enhance C_{bb} , C_{qq} and contribute to C_{bq} with the same sign as the associated particle. It is important to note that C_{bb} and C_{qq} depend on b_j^2 and q_j^2 . They are measures of baryon number and charge fluctuations and depend on the fundamental baryon number and electric charge. In a Q-g phase these coeffi-

icients will be different since the unit of charge is $1/3$ rather than 1 as used in Refs. [1, 11, 12]. Here, the focus will be properties of the hadron phase and the rise of the specific heat as it approaches a possible transition temperature T_0 . In a future paper a discussion of the specific heat of the Q-g phase will be given. It can be calculated in the simple approximation of ideal gases of gluons and quarks using $T\partial S/\partial T$ where S is the entropy. For the case of massless pions and all other mesons and baryons taken in the non-relativistic limit, the results of Eq. (6) become

$$(7) \quad \begin{aligned} f_B &= \frac{-(3B/2 + M_B/T)C_{qq}}{(3Z/2 + 3Z_\pi/2 + M_Q/T)C_{bq}}/C \\ f_Q &= \frac{-(3Z/2 + 3Z_\pi/2 + M_Q/T)C_{bb}}{(3B/2 + M_B/T)C_{bq}}/C \end{aligned}$$

The $Z_\pi = N_{\pi^+} - N_{\pi^-}$, while the Z is the total conserved charge and also contains the contribution of the charged pions. However, massless pions do not affect M_Q . For non-relativistic pions, the contribution of pions appears in M_Q and the extra Z_π term in Eq. (7) is no longer present.

Expressions for the specific heat of hadronic matter

The specific heat of hadronic matter in the non-degenerate and the non-relativistic limit for all particles is given by

$$(8) \quad \begin{aligned} C_V &= \left(\frac{3}{2}\right) \sum_j N_j + \sum_j \left(\frac{3}{2} + \frac{m_j}{T}\right)^2 N_j \\ &+ \left\{ -\left(\frac{3B}{2} + \frac{M_B}{T}\right)^2 C_{qq} - \left(\frac{3Z}{2} + \frac{M_Q}{T}\right)^2 C_{bb} \right. \\ &+ \left. 2\left(\frac{3B}{2} + \frac{M_B}{T}\right)\left(\frac{3Z}{2} + \frac{M_Q}{T}\right)C_{bq} \right\} / C \\ &= \left(\frac{3}{2}\right) \sum_j N_j + \sum_j \left(\frac{3}{2} + \frac{m_j}{T}\right)^2 N_j - C_{bb}(f_B)^2 - C_{qq}(f_Q)^2 \end{aligned}$$

The first term on the right hand side of the last equation is just the ideal gas specific heat of each non-relativistic particle, with both mesons and baryons contributing. The second term involves the mass spectrum of all particles produced. The curly bracket or third term in the first equality has three contributions and involves the three coefficients C_{bb} , C_{qq} and C_{bq} . The second and the third term arise from the possibility that the particle distributions change with T or $\partial N_j/\partial T \neq 0$. The second term will be cancelled by the third term for a system which has $\partial N_j/\partial T = 0$, for all j . Specifically, consider a system of N_p protons and N_n neutrons, so that $N_p = Z$ and $N_p + N_n = B$. Then $C_{bb} = B$, $C_{qq} = Z$ and $C_{bq} = Z$. When these results are substituted into Eq. (10), along with the results $M_B = m_N B$ and $M_Q = m_N Z$, the second and the third term exactly cancel. The specific heat then reduces to the ideal gas law $C_V = (3/2)B$. The last equality replaces the curly bracket term with the functions

$-C_{bb}(f_B)^2 - C_{qq}(f_Q)^2$ (and connects C_V to information about the behavior of the baryon and electric charge chemical potentials. When pions are taken in the zero mass and non-degenerate limit, the C_V is somewhat modified and now reads

$$(9) \quad \begin{aligned} C_V &= \left(\frac{3}{2}\right) \sum_j N_j + \sum_j \left(\frac{3}{2} + \frac{m_j}{T}\right)^2 N_j + 12(N_{\pi^+} + N_{\pi^0}) \\ &+ \left\{ -\left(\frac{3B}{2} + \frac{M_B}{T}\right)^2 C_{qq} - \left(\frac{3Z}{2} + \frac{3Z_\pi}{2} + \frac{M_Q}{T}\right)^2 C_{bb} \right. \\ &+ \left. 2\left(\frac{3B}{2} + \frac{M_B}{T}\right)\left(\frac{3Z}{2} + \frac{3Z_\pi}{2} + \frac{M_Q}{T}\right)C_{bq} \right\} / C \end{aligned}$$

The first two sums over j on the right hand side of this equation exclude the pion contribution in their evaluation. The pion contribution is now contained in the following terms in that equation. The third term is the direct contribution of the pion as if it were independent of the charge conservation law and the curly bracket term arises from the chemical potentials and associated constraints. When these constraints are neglected, the curly bracket term is zero. The independent pion contribution can also be calculated using the results of Eq. (1) and Eq. (2). The exact expression for the specific heat per particle of an unconstrained meson or boson including statistical corrections reads:

$$(10) \quad \frac{C_{V,m}}{N_m} = (m/T)^2 \cdot \frac{\sum_k K_2(km/T) + 3(T/m)K_3(km/T)/k}{\sum_k K_2(km/T)/k}$$

The $m \rightarrow 0$ limit of this equation is $C_{V,m}/N_m = 12\zeta(4)/\zeta(3)$. If statistical corrections are neglected, this limit would be 12 , with the zeta functions giving the corrections from the sums over k in Eq. (10). The non-degenerate and large m/T limit of Eq. (10) is

$$(11) \quad C_{V,m}/N_m = (3/2 + m/T)^2 + 3/2$$

This is the characteristic dependence of the first 2 terms in Eq. (8).

Simplified model I; only conserved baryon charge

To gain some further insight into properties of C_V , a simplified situation of one conserved charge will be considered. Namely, in this subsection only baryon number conservation will be imposed on the system. Then, all charge and neutral states of the same baryon will have equal yields. Mesons and baryons will also completely decouple and the specific heat C_V will be a sum of independent contributions from mesons given by Eq. (10) and constrained baryons. For this model, the baryon constraint of Eq. (4) is $3B/2 + M_B/T + C_{bb}f_B = 0$ and the contribution of the baryons to the specific heat, $C_{V,B}$:

$$(12) \quad C_{V,B} = \frac{3}{2} \sum_j N_{j,B} + \sum_j \left(\frac{3}{2} + \frac{m_j}{T} \right)^2 N_{j,B} - \left(\frac{1}{C_{bb}} \right) \left(\frac{3B}{2} + \frac{M_B}{T} \right)^2$$

The sums over j are over both baryons and antibaryons. Antibaryons are usually a small fraction of the total baryon number. If we allow only baryons with $b_j = 1$, then $C_{bb} = B$, and Eq. (17) can be rewritten in a simpler form

$$(13) \quad C_{V,B} = \frac{3}{2}B + B \frac{1}{T^2} \left(\langle m_B^2 \rangle - \langle m_B \rangle^2 \right)$$

where the mean mass and its fluctuation are determined by

$$(14) \quad \langle m_B \rangle = \sum_j m_j N_{j,B} / B, \quad \langle m_B^2 \rangle = \sum_j m_j^2 N_{j,B} / B$$

Thus, the enhancement to the ideal gas specific heat $C_{V,B} = (3/2)B$ involves the mean square fluctuation in the resonance mass excitation. The f_B can be used to obtain the mean baryonic mass that is excited by rearranging the baryon constraint condition to read

$$(15) \quad \langle m_B \rangle = -3T/2 + (C_{bb}/B)(\mu_B = \partial\mu_B/\partial T)$$

Again, if antibaryon production is neglected, the coefficient $C_{bb}/B = 1$. The condition $\partial^2 B/\partial T^2 = 0$ can be used to obtain an expression for the mean square fluctuation in the masses that are excited. This condition and the case for all $b_j = 1$ gives

$$(16) \quad \langle m_B^2 \rangle - \langle m_B \rangle^2 = -3T^2/2 - T^3 \partial^2 \mu_B / \partial T^2$$

Using this last result, the $C_{V,B}$ is simply

$$(17) \quad C_{V,B} = -T \partial^2 \mu_B / \partial T^2 = 3/2 + (\delta m_B)^2 / T^2$$

Role of antibaryons

The presence of antibaryons will modify some of the results given in the previous Section. For collision energies ≤ 100 AGeV antibaryons make up a few percent of B . From Ref. [2], the antiproton, proton ratio is $\sim 2\%$ for the 80 AGeV Pb+Pb collision. This ratio rises to $\sim 5\%$ for the 158 AGeV collision. For a $\mu_B \sim 300$ MeV and $T \sim 150$, $\exp(-2\mu_B/T) = \exp(-4) \sim 2\%$, which determines the antiparticle/particle ratio in the absence of an electric chemical potential. The antiparticle, particle ratio will increase at much higher energy because μ_B decreases and T increases. The presence of both antibaryons with fraction $y = N_{\bar{B}}/(N_B + N_{\bar{B}})$ and baryons with fraction $x = N_B/(N_B + N_{\bar{B}})$ leads to a modified form for the baryonic (plus antibaryonic) $C_{V,B}$:

$$(18) \quad C_{V,B}/(N_B + N_{\bar{B}}) = 3/2 + (x(\delta m_B)^2 + y(\delta m_{\bar{B}})^2)/T^2 + xy(3 + (\langle m_B \rangle + \langle m_{\bar{B}} \rangle)/T)^2 = 3/2 + (\delta m_B)^2/T^2 + 4xy(3/2 + \langle m_B \rangle/T)^2$$

The last equality arises from $\langle m_B \rangle = \langle m_{\bar{B}} \rangle$ and $\delta m_B = \delta m_{\bar{B}}$. The specific heat is also

$$(19) \quad C_{V,B}/(N_B + N_{\bar{B}}) = -((N_B + N_{\bar{B}})/B) (T \partial^2 \mu_B / \partial T^2) + 4(N_B N_{\bar{B}}/B^2) f_B^2 = -\coth(\mu_B/T) T \partial^2 \mu_B / \partial T^2 + 2 \operatorname{csch}^2(\mu_B/T) (\partial \mu_B / \partial T - \mu_B/T)^2$$

When $N_{\bar{B}} \rightarrow 0$, or μ_B/T very large, the r.h.s. of Eqs. (18, 19) reduces to Eq. (17). The specific heat now involves both the curvature and slope of the chemical potential and the value of the chemical potential itself. In the limit $x = y = 1/2$ the $C_{V,B}$ of Eq. (18) becomes the unconstrained limit: $C_{V,B}/(N_B + N_{\bar{B}}) = 3/2 + (9/4 + 3\langle m_B \rangle/T + \langle m_B^2 \rangle/T^2)$. The ideal gas limit $C_{V,B}/B = 3/2$ is realized in the limit $x = 1, y = 0$ and $\delta m_B = 0$. The $\langle m_B \rangle$ and $\langle m_B^2 \rangle - \langle m_B \rangle^2 = (\delta m_B)^2$ are given by:

$$(20) \quad \langle m_B \rangle/T = -3/2 - \coth(\mu_B/T) (\partial T - \mu_B/T) (\delta m_B)^2/T^2 = -3/2 - \coth(\mu_B/T) \cdot (T \partial^2 \mu_B / \partial T^2) + \operatorname{csch}^2(\mu_B/T) (\partial \mu_B / \partial T) (\partial \mu_B / \partial T - \mu_B/T)^2$$

Thus, the values of μ_B , its derivative and curvature also contain the information necessary to evaluate various quantities of interest regarding the mass excitation.

Role of electric charge conservation

A hybrid case consists of no antibaryon production (all $b_j = 1$), and having both baryon and electric charge conservation. This model reflects the fact that antibaryon production is suppressed compared to \pm charged particle production. The production of \pm charged pion pairs is easier than a baryon-antibaryon pair. This hybrid model is also useful as a way of seeing how the two constraints act together in the expression for the specific heat and mass spectrum of produced hadrons. To keep final results as simple as possible within this hybrid case, the assumptions $\langle m_{B,+Q} \rangle = \langle m_{B,-Q} \rangle = \langle m_{B,0} \rangle$ and $\langle m_{M,+Q} \rangle = \langle m_{M,-Q} \rangle$ will be made. Here, the subscripts ($B,+Q$) refer to baryons with charge $+Q$, ($M,+Q$) to mesons with $+Q$, etc. The uncharged mesons, ($M,0$), are decoupled from the baryon and electric charge conservation conditions and consequently add to C_V independently. C_V in this case reads

$$(21) \quad C_V = \frac{3}{2} (B + N_M) + \left(\frac{3}{2} + \frac{\langle m_{B,+Q} \rangle}{T} \right)^2 B + \left(\frac{3}{2} + \frac{\langle m_{M,+Q} \rangle}{T} \right)^2 N_M + \frac{B \cdot \delta m_B^2 + N_M \cdot \delta m_M^2}{T^2} - B f_B^2 - (N_+ + N_-) f_Q^2$$

The $B = N_{B,+Q} + N_{B,-Q} + N_{B,0}$, $N_M = N_{M,0} + N_{M,+Q} + N_{M,-Q}$, N_{\pm} is the total \pm charge in mesons and baryons or $N_{\pm} = N_{M,\pm Q} + N_{B,\pm Q}$ and $N_+ + N_- = N_{CB} + N_{CM}$ with N_{CB} and N_{CM} the total number of charged baryons and charged mesons, respectively. The mean baryon mass and mean meson mass are given by

$$(22) \quad \begin{aligned} \langle m_{B,+Q} \rangle / T &= -3/2 - f_B - (Z_B/B) f_Q \\ \langle m_{M,+Q} \rangle / T &= -3/2 - (1/Z_M) \\ &\quad \cdot (N_+ + N_- - (Z - Z_M)^2/B) f_Q \end{aligned}$$

The $Z = Z_M + Z_B$, where $Z_M = N_{M,+Q} - N_{M,-Q}$ and $Z_B = N_{B,+Q} - N_{B,-Q}$. Also

$$(23) \quad (\delta m_{B,+Q})^2 / T^2 = -3/2 - T \partial \mu_B / \partial T^2 - (Z_B/B) T \partial^2 \mu_Q / \partial T^2 - (N_{B,+Q} + N_{B,-Q} - (Z_B^2/B)) f_Q^2$$

for the mean square fluctuation for baryons and the mean square fluctuation for mesons is

$$(24) \quad Z_M (\delta m_M)^2 / T^2 = -3Z_M/2 - (N_+ + N_- - Z_B^2/B) \cdot T \partial^2 \mu_Q / \partial T^2 - a_Q f_Q^2$$

with a_Q given by

$$(25) \quad a_Q = Z - 3Z_B N_{CB}/B + Z_B^3/B^2 + (1/Z_M)((N_{CB} - Z_B^2/B)^2 - N_{CM}^2)$$

The specific heat can then be obtained from the above equations when they are substituted into Eq. (21). The result relates C_V to properties of μ_B and μ_Q . The curvature of μ_B and μ_Q appear in expressions for the mass fluctuation, which also involves the slope and value of these functions. The mean excited meson mass only involves properties of μ_Q such as its slope and value, while the mean excited baryon mass involves similar properties of both μ_B and μ_Q .

Some features of the statistical model analysis of CERN/RHIC data

The result of the previous sections can be used to evaluate some features of recent CERN/RHIC data. In this Ref. [2], the T dependence of μ_B is given in an equation that reads $T = .167 - .153\mu_B^2$. The T and μ_B are in GeV. Here, a more general form for the T dependence of μ_B is used, namely $T = T_0 - a(\mu_B)^{1/\beta}$.

Substituting this result into Eq. (21) and Eq. (23) gives for $T \rightarrow T_0$

$$(26) \quad \begin{aligned} \langle m_B \rangle / T &\rightarrow \beta T_0 / (T_0 - T) \\ (\delta m_B)^2 / T^2 &\rightarrow \beta T_0^2 / (T_0 - T)^2 \\ C_{V,B} / (N_B + N_{\bar{B}}) &\rightarrow \beta(\beta + 1) T_0^2 / (T_0 - T)^2 \end{aligned}$$

Thus, β does not appear as an exponent in the divergence of the specific heat which is quadratic or $1/(T_0 - T)^2$ for all β . The $\langle m_B \rangle \rightarrow \infty$ as $T \rightarrow T_0$. Using values for μ_B and T given in Ref. [2] for Pb+Pb collisions at 30, 40, 80 and 158 A GeV and for Au+Au 11.6 A GeV collisions, and the above parameterization of the behavior of μ_B with T with $\beta = 1/2$, the values of $\langle m_B \rangle$, $(\langle m_B^2 \rangle - \langle m_B \rangle^2)^{1/2} \equiv \delta m_B$ in MeV and $C_{V,B}/B$ are shown in Table 1. The last three columns are obtained from results in the previous Section that include antibaryons. The previous three columns are without antibaryons obtained from expressions in the section ‘Simplified model 1; only conserved baryon charge’. The curvature and slope functions are obtained from this parameterization. The antibaryon case also used this parameterization to evaluate the chemical potential, while the case with just baryons used the chemical potential of Ref. [2]. The error bars in T and μ_B are not given and generate large error bars in the results for the mean mass, mass fluctuation and specific heat, especially at the higher temperatures. These errors are typically $\pm 20\%$. Two sets of numbers for each energy appear in the table since Ref. [2] has two main analysis of the data, called *A* and *B*. The results presented in the table show that the specific heat per particle for baryons are very different from the ideal gas contribution of 1.5.

The contribution of decoupled mesons to C_V can be obtained from Eq. (12). In Ref. [2]: π 's = 1356, K 's = 160, K^* 's = 66, ρ 's = 149, η = 49, ω = 40, are the multiplicity yields for the 80 GeV*A Pb+Pb collision for the low lying well known mesons. Using Eq. (12), each of these mesons contributes to C_V as follows: $C_{V,\pi}/N_\pi = 13.2$, $C_{V,K} = 28.8$, $C_{V,\eta}/N_\eta = 32$, $C_{V,\omega}/N_\omega = 49.5$, $C_{V,\rho}/N_\rho = 48.5$, $C_{V,K^*}/N_{K^*} = 59$. Thus mesons make a very large contribution to C_V also.

Table 1. Values of the specific heat per particle, mean baryonic mass $\langle m_B \rangle$ and mass fluctuation δn_B in MeV excited in various collisions. Main analysis *A* is the first set of numbers in each row, main analysis *B* is the second set of numbers

Energy [MeV]	T	μ_B	$\langle m_B \rangle$	δn_B	$C_{V,B}/B$	$\langle m_B \rangle$	δn_B	$\frac{C_{V,B}}{(N_B + N_{\bar{B}})}$
11.6	118.1	555	1061	277	7.0	1071	277	7.0
	119.1	578	1094	286	7.3	1086	286	7.3
30	139.5	428.6	1295	593	19.6	1277	594	20.7
	140.3	428	1316	612	20.6	1313	614	21.9
40	147.6	380.3	1513	853	34.9	1517	860	39.9
	145.5	375.4	1426	770	29.5	1444	775	32.8
80	153.7	297.7	1771	1216	64.0	1856	1243	82.1
	151.7	288.9	1629	1070	51.2	1715	1087	62.7
158	157.8	247.3	2113	1676	114.3	2331	1753	168.4
	154.8	244.5	1804	1313	73.4	1952	1348	97.0

Summary and conclusions

The properties of the specific heat of hadronic matter produced in very high energy nucleus-nucleus collisions such as those at RHIC and CERN are studied in this paper.

The grand canonical statistical model is used to develop expressions for the particle multiplicity distribution and energy caloric equation of state which is then used to obtain the specific heat. The constraints associated with baryon number and electric charge conservation are included to obtain an expression for the specific heat which contains the particle yields, the mass spectrum of produced particles, baryon number B and overall charge Z values, and three coefficients C_{bb} , C_{qq} , and C_{bq} . These coefficients come from the correlations introduced by requiring overall baryon number and electric charge conservation in the spectrum of produced particles. The coefficients also depend on the fundamental unit of charge being different for the case of $1/3$ (Q-g phase) compared to 1 (hadron phase). C_{bb} and C_{qq} are also measured of baryon number and charge fluctuations [1, 11, 12]. The specific heat is not simply a sum of independent contributions arising from each type of particle. Rather C_V has additional terms which significantly alter its value from this independent particle result. Resonance excitations allow for the possibility that individual particle yields change with T and redistribute the conserved charge and baryon number on other particles. The behavior of C_V is studied in some limiting cases to see how various quantities such as baryonic charge conservation, the production of anti-baryons and electric charge conservation affect it. The mass fluctuation in excited resonances is shown to play an important role. Using properties of the constraint equations, the specific heat and mass spectrum of excited hadrons is related to properties of the baryon and electric charge chemical potentials μ_B and μ_Q . In particular, the specific heat is related to the curvature of the chemical potentials, its slope and its value. The mean hadronic mass that is produced in a heavy-ion collision involves the chemical potentials and their slopes, and the mass fluctuation involves these quantities and the curvature of the chemical potentials. A recent parameterization of the baryonic chemical potential with T is shown to lead to a very rapid increase in the baryonic component of the specific heat. If this expression correctly describes the behavior of μ_B near a limiting temperature T_0 , then C_V would diverge as $1/(T_0 - T)^2$. Moreover, the exponent 2 is independent of the functional form used near T_0 . The presence of antibaryons plays a key role in the temperature behavior of C_V , and this independence property.

Acknowledgments The author would like to thank the DOE for its support of this research under grant number DOE: FG02-96ER-40987.

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