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PROCEEDINGS

Freeze-out process with in-medium nucleon mass

Sven Zschocke,
Laszlo P. Csernai,
Etele Molnár,
Jaakko Manninen,
Agnes Nyiri

Abstract We investigate the kinetic freeze-out scenario of a nucleon gas through a finite layer. The in-medium mass modification of nucleons and its impact on the freeze-out process is studied. A considerable modification of the thermodynamical parameters temperature, flow-velocity, energy density and particle density has been found in comparison with evaluations which use a constant vacuum nucleon mass.

Key words freeze-out • nucleon mass shift • chiral condensate • heavy-ion collision

S. Zschocke✉
Section for Theoretical and Computational Physics,
and Bergen Computational Physics Laboratory,
University of Bergen,
5007 Bergen, Norway
and Forschungszentrum Rossendorf,
01314 Dresden, Germany,
Tel.: +49-351-2603263, Fax: +49-351-2603700,
E-mail: zschocke@fz-rossendorf.de

L. P. Csernai
Section for Theoretical and Computational Physics,
and Bergen Computational Physics Laboratory,
University of Bergen,
5007 Bergen, Norway
and MTA-KFKI, Research Institute
of Particle and Nuclear Physics,
1525 Budapest 114, Hungary

E. Molnár, A. Nyiri
Section for Theoretical and Computational Physics,
and Bergen Computational Physics Laboratory,
University of Bergen,
5007 Bergen, Norway

J. Manninen
Section for Theoretical and Computational Physics,
and Bergen Computational Physics Laboratory,
University of Bergen,
5007 Bergen, Norway
and University of Oulu,
Department of Physical Sciences,
90571 Oulu, Finland

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Introduction

As the fireball of a nucleus-nucleus collision cools down below a certain freeze-out temperature $T_{\mathrm{FO}} \leq T_c \simeq 175$ MeV the inelastic (chemical freeze-out) and elastic (kinetic freeze-out) collisions among the hadrons cease. This process is usually called the freeze-out scenario (FO). Several approaches have been applied for the description of the freeze-out of strongly interacting matter. Especially, there are kinetic models [1, 11] as well as hydrodynamical approaches [5] which have been proven to be able to describe most of the collective phenomena like the different flow components in heavy-ion reactions. Despite the success in comparison with experiments, the in-medium modifications of the hadrons during the freeze-out process have not been taken into account yet. In all of the former evaluations the vacuum parameters of the particles have been implemented. However, during the freeze-out process, the temperature and particle densities are presumably close to the deconfinement phase transition critical values [2]. Accordingly, strong in-medium modifications of hadronic properties like mass, width, coupling constants, etc., are expected. The question arises how strong their impact on the freeze-out process is. For answering this issue, we study a nucleon gas and investigate how strong is the impact of an in-medium nucleon mass shift on the thermodynamical parameters, temperature, flow-velocity, energy density and particle density, of the freeze-out scenario.

Freeze-out process within a finite layer

The theoretical description of the kinetic freeze-out within a hydrodynamical approach has been worked out some years ago (see [8] and references therein). Here, the evaluations presented are based on the approach of Ref. [9], where the freeze-out description has been generalized to the case of a finite time-like layer (a finite space-like layer has also been investigated in [10]; more details about the approach presented will be published in [12]). Accordingly, local equilibrium implies that the thermodynamical parameters inside the layer become space-time dependent, i.e. we have a space-time dependent temperature $T(x)$, flow velocity $v(x)$, energy density $e(x)$ and nucleon density $n(x)$. From the equations of hydrodynamics, we derive the following set of coupled differential equations for three unknowns

$$(1)\quad de(x) = u_\mu(x)dT^{\mu\nu}(x)u_\nu(x) + 2du_\mu(x)T^{\mu\nu}(x)u_\nu$$

$$(2)\quad dn(x) = u_\mu(x)dN^\mu(x)$$

$$(3)\quad du^\mu(x) = \frac{1}{n(x)}\left(g^{\mu\nu} - u^\mu(x)u^\nu(x)\right)dN_\nu$$

The differentials in Eqs. (1)–(3) are deduced from the microscopic definition, $dN^\mu(x) = \int[(d^3k)/k^0]k^\mu df(x,k)$ for the particle current, and $dT^{\mu\nu} = \int[(d^3k)/k^0]k^\mu k^\nu df(x,k)$ for the energy momentum tensor. Here, $x^\mu = (t,\mathbf{r})$ is the four-coordinate and $k^\mu = (E_k,\mathbf{k})$ is the four-momentum of the nucleon; the scalar one-particle distribution function is normalized to the invariant number of nucleons N, i.e. $N = \int d^3\mathbf{r}d^3\mathbf{k}f(x,k)$. We mention that the second term in Eq. (1) vanishes within the approach presented, but not in general. The rest frame of the gas (RFG) is defined by $u^\mu_{\mathrm{RFG}}(x) = (1,0,0,0)$. For baryon-dominated matter, like in our case, the Eckart's definition [3] of four-flow is commonly used, defined by $u^\mu(x) = N^\mu(x)/\sqrt{N^\nu(x)N_\nu(x)}$. Another Lorentz frame, e.g. rest frame of the front (RFF), can be defined by a Lorentz boost in respect to RFG.

Since there are four unknowns in the problem under consideration an additional constraint is necessary, which is provided by the equation of state (EOS) for the nucleon gas [7]

$$(4)\quad e(x) = n(x) \cdot\left[M_N\left(n(x),T(x)\right) - E_0 + \frac{K}{18}\left(\frac{n(x)}{n_0} - 1\right)^2 + \frac{3}{2}T(x)\right]$$

The nuclear binding energy is E_0 = 16 MeV, and K = ≃ 235 MeV is the compressibility; $M_N(n,T)$ is the in-medium nucleon pole mass. The EOS (Eq. (4)) is used to determine the temperature $T(x)$ of the interacting component of the nucleon gas during the freeze-out process. Accordingly, the four equations (1), (2), (3) and (4) represent a closed set for evaluating the four unknowns T,v,e,n of the one-particle system.

Furthermore, as the system expands and cools down the number of interacting particles decreases up to the post freeze-out surface of the finite layer, where by definition the density of interacting particles vanishes. Correspondingly, the particle distribution function is decomposed into two components, an interacting part f_i and a non-interacting part f_f, thus $f(x,k) = f_i(x,k) + f_f(x,k)$. Accordingly, there is an interacting particle density n_i and a non-interacting particle density n_f with $n = n_f + n_i$. On the pre-freeze out hypersurface we assume to have thermal equilibrium, i.e. we have a Jüttner distribution, cf. [3], for f_i as starting a one-particle distribution function, while by definition f_f is zero on the pre-freeze out hyper-surface. The space-time evolution of the interacting and non-interacting components inside the layer is governed by the following differential equations [9]:

$$(5)\quad \partial_t f_i = -\frac{1}{\tau}\left(\frac{L}{L-t}\right)\left(\frac{k^\mu d\sigma_\mu}{k_\mu u^\mu}\right)f_i + \frac{1}{\tau_0}\left[f_{\mathrm{eq}}(t) - f_i\right]$$

$$(6)\quad \partial_t f_f = +\frac{1}{\tau}\left(\frac{L}{L-t}\right)\left(\frac{k^\mu d\sigma_\mu}{k_\mu u^\mu}\right)f_i$$

with the time τ between collisions, and f_{eq} is the Jüttner distribution, cf. [3]. The second term in Eq. (5) is the re-thermalization term [8], which describes how fast the interacting component approaches the Jüttner distribution within a relaxation time τ_0. Here, we will use the immediate re-thermalization limit $\tau_0 \to 0$, which implies $f_i \to f_{\mathrm{eq}}$ faster than $\tau_0 \to 0$. The explicit expressions for the differentials dN^μ and $dT^{\mu\nu}$ within the approach presented can be found in [9].

The set of equations (1)–(6) allow us to evaluate the basic thermodynamical function $T(x)$, $v(x)$, $e(x)$ and $n(x)$ during the freeze-out process for a particle with a mass $M_N(n,T)$.

Nucleon mass shift

A generalization of the QCD sum rule approach for nucleons at finite densities and temperatures [4] leads to the following expression for the pole mass of a nucleon embedded in a hot and dense hadronic medium,

$$(7)\quad M_N(n,T) = M_N(0) + \mathrm{Re}\Sigma_S(n,T) + \mathrm{Re}\Sigma_V(n,T)$$

where $M_N(0)$ = 939 MeV is the vacuum nucleon pole mass with the attractive scalar part ($\mathrm{Re}\Sigma_S < 0$) and the repulsive vector part ($\mathrm{Re}\Sigma_V > 0$) of nucleon self energy in medium. Typical values at nuclear saturation density n_0 = 0.17 fm^{-3} and at vanishing temperature are $\mathrm{Re}\Sigma_S$ = –400 MeV, $\mathrm{Re}\Sigma_V$ = +300 MeV [4]. Here, we will take the QCD sum rule results for a nucleon in matter, given by [4]

$$(8)\quad \mathrm{Re}\Sigma_S(n,T) = +M_N(0)\left(\frac{\langle\Omega|\bar{q}q|\Omega\rangle}{\langle 0|\bar{q}q|0\rangle} - 1\right)$$

$$(9)\quad \mathrm{Re}\Sigma_S(n,T) = -\frac{8}{3}M_N(0)\frac{\langle\Omega|q^\dagger q|\Omega\rangle}{\langle 0|\bar{q}q|0\rangle}$$

Here, $\langle\Omega|\bar{q}q|\Omega\rangle$ and $\langle\Omega|q^\dagger q|\Omega\rangle$ are in-medium condensates and $|\Omega\rangle$ is a state which describes the hot and dense hadronic matter inside the layer, while $\langle 0|\bar{q}q|0\rangle = (-0.250\ \mathrm{GeV})^3$ is the chiral condensate. In Eqs. (8) and (9) we have neglected the gluon condensate and higher mass dimension condensates which give rise to small corrections only.

There are two nucleonic components inside the finite layer: an interacting component with density n_i and a non-interacting component with density n_f. For evaluating the condensates Eqs. (8) and (9) we approximate the interacting component by a Fermi gas with chemical potential μ_i and temperature T. On the other side, the temperature for the non-interacting component becomes ill-defined. Nonetheless, a relevant physical parameter for describing the non-interacting component remains the density n_f. Accordingly, the condensates in one-particle approximation are given as follows [13]:

$$(10)\quad \langle\Omega|\bar{q}q|\Omega\rangle = \langle 0|\bar{q}q|0\rangle + \hat{I}(\mu_i,T)\langle N(\mathbf{k})|\bar{q}q|N(\mathbf{k})\rangle + \frac{n_f}{2M_N(0)}\langle N(\mathbf{k})|\bar{q}q|N(\mathbf{k})\rangle$$

$$(11)\quad \langle\Omega|q^\dagger q|\Omega\rangle = \hat{I}(\mu_i,T)\langle N(\mathbf{k})|q^\dagger q|N(\mathbf{k})\rangle + \frac{n_f}{2M_N(0)}\langle N(\mathbf{k})|q^\dagger q|N(\mathbf{k})\rangle$$

with $\hat{I} \equiv 4\int d^3\mathbf{k}/[(2E_k)(2\pi)^3(\exp((E_k-\mu_i)/T)+1)]$; for vanishing temperature we have $I \to n_i/(2M_N)$. Note that $\langle 0|q^\dagger q|0\rangle = 0$; the nucleon energy is $E_k = \sqrt{M_N(0)^2 + \mathbf{k}^2}$. For nucleons we take the relativistic normalization $\langle N(\mathbf{k}_1)|N(\mathbf{k}_2)\rangle = 2E_{k1}(2\pi)^3\sigma^{(3)}(\mathbf{k}_1 - \mathbf{k}_2)$ is used. The chemical potential for the interacting component can be evaluated via $n_i = 4\int d^3\mathbf{k}/[(2\pi)^3(\exp(E_k - \mu_i)/T + 1)]$. The condensates in Fermi gas approximation are given by [6] $\langle N(\mathbf{k})|\bar{q}q|N(\mathbf{k})\rangle = M_N(0)\sigma_N/m_q$ and $\langle N(\mathbf{k})|q^\dagger q|N(\mathbf{k})\rangle = 3M_N(0)$. The nucleon sigma term is $\sigma_N \simeq 50$ MeV, and $m_q \simeq 5$ MeV is the averaged current quark mass of the light quarks. Inserting these parameters into Eqs. (8) and (9), we obtain $\mathrm{Re}\Sigma_S = -390$ MeV and $\mathrm{Re}\Sigma_V = 315$ MeV at ground state saturation density n_0. Equations (7)–(11) summarize our propositions made for obtaining the in-medium nucleon pole mass $M_N(n,T)$, which enters the EOS (4) and the differentials dN^μ and $dT^{\mu\nu}$.

Results and discussion

For all of the calculations, we have taken $T_{\mathrm{preFO}} = 150$ MeV, $n_{\mathrm{preFO}} = 1.5n_0$, and $n_{\mathrm{preFO}} = 0.5c$, as starting values on the pre-freeze out hypersurface. These values are, for instance, in line with typical parameters which have been reached within the Alternating-Gradient Synchrotron (AGS) at Brookhaven National Laboratory (BNL) in Brookhaven/USA. Higher baryonic densities can be reached within the Schwer-Ionen-Synchrotron (SIS) at Gesellschaft für Schwerionenforschung (GSI) in Darmstadt/Germany. Note that T_{preFO} and n_{preFO} are pre-freeze out values and, therefore, they are larger than typical post-freeze out values given, for instance, in Ref. [2].

In Figs. 1 and 2, the time evolution of the primary thermodynamical functions through the finite freeze-out layer are shown, in terms of the proper time τ. Note that the densities $n = n_i + n_f$ and $e = e_i + e_f$ are kept constant inside the layer.

We find a substantial impact of in-medium mass modification on the freeze-out process within the purely nucleon gas model. Furthermore, Figs. 1 and 2 also elucidate that the freeze-out process proceeds faster for all thermodynamical quantities T,v,e,n when taking into account the mass dropping of nucleons. The physical reason for a faster freeze-out originates from a smaller energy density of the nucleon system due to a smaller nucleon mass $M_N(n,T)$ compared to the vacuum nucleon mass $M_N(0)$.

Finally, we remark that in-medium modifications have actually to be taken into account not only during the freeze-out process, but also before, i.e. during the hadronization. This points then to an even stronger impact of in-medium modifications on the final particle spectrum than presented.

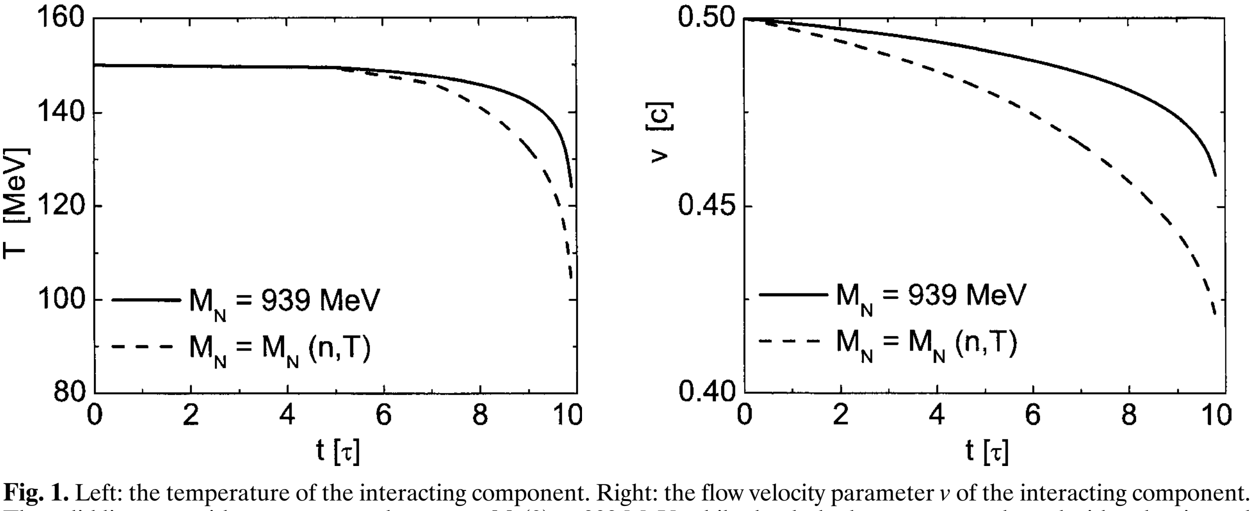

Fig. 1. Left: the temperature of the interacting component. Right: the flow velocity parameter v of the interacting component. The solid lines are with a constant nucleon mass $M_N(0) = 939$ MeV, while the dashed curves are evaluated with a density and temperature dependent nucleon mass $M_N(n,T)$.

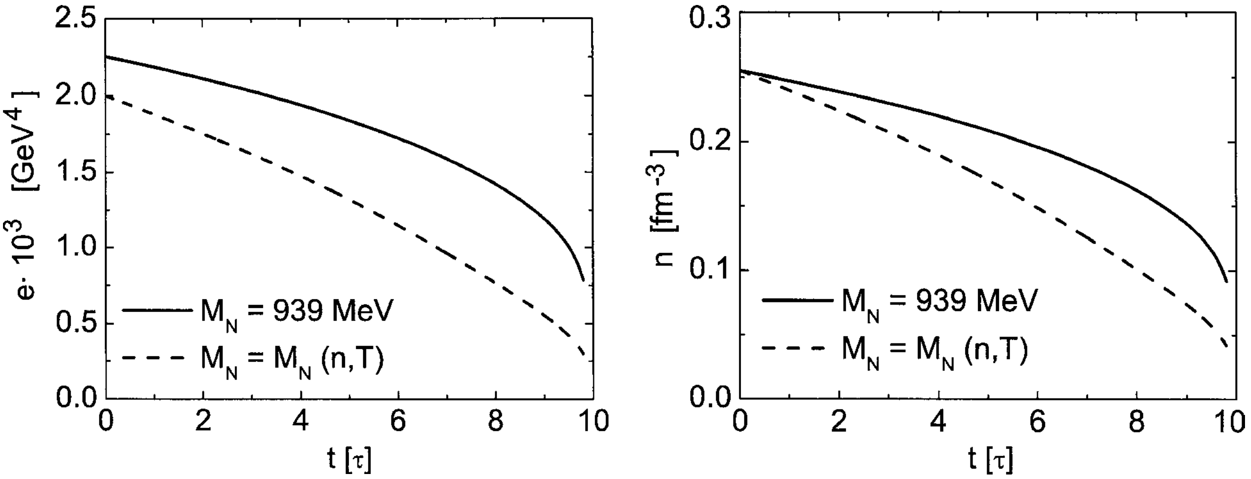

Fig. 2. Left: nucleon energy density of the interacting component. Right: nucleon particle density of the interacting component. The solid lines are with a constant nucleon mass $M_N(0) = 939$ MeV, while the dashed curves are evaluated from the density and temperature dependent nucleon mass $M_N(n,T)$.

Summary

We have investigated a freeze-out scenario within a finite layer for a massive nucleon gas. Special attention has been drawn to the issue how strong is the impact of the in-medium nucleon mass modification on the thermal freeze-out process. By focussing on a purely nucleon gas, we have found a substantial effect on the thermodynamical quantities like temperature T, flow velocity v, particle density n and energy density e of the interacting component. All of these thermodynamical functions have revealed a faster freeze-out compared to a scenario without an in-medium nucleon mass shift.

In summary, our findings for a nucleon gas suggest that taking into account in-medium modifications of nucleons seems to be necessary and is an interesting phenomenon, in particular, for collision scenarios with high baryonic densities.

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