Alternative size and lifetime measurements for RHIC

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Abstract Two-particle correlations based on the interference of identical particles have provided the chief means for determining the shape and lifetime of sources in relativistic heavy ion collisions (RHIC). Here, strong and Coulomb induced correlations are shown to provide similar information.

Key words correlations • interferometry • identical particles • heavy ion collision

In this workshop, we have seen remarkable evidence of the rapid progress of two-pion and two-kaon correlations with regards to determining the space-time characteristics of emitting sources in collisions from RHIC. These correlations are driven by the identical-particle interference of two bosons [10]. Over the last twenty years, the measurements and associated phenomenology have increased in their sophistication to the point where all six dimensions of the correlations are currently being exploited as can be seen in [17].

The results from two-pion correlations at RHIC [1, 2, 4] have shattered expectations [8, 21, 24, 25] of long-lived sources which should have resulted from the softening of the equation of state accompanying the deconfinement transition. Hydrodynamic calculations that incorporate lattice-gauge inspired equations of state and breakup criteria consistent with expected hadronic cross sections predict sources which stay together for times of 15–20 fm/c with continuous emission during the transverse expansion. Instead, two-pion correlations [17, 18] are consistent with sources which disassociate at ~10 fm/c in a sudden flash [6, 17]. Even more remarkably, the transverse size of the system appears to be near 13 fm, which would require an extremely rapid acceleration of the system which begins life with a transverse radius of ~6 fm. Although surface velocities indeed appear to be of the order of 0.7 c in blastwave fits to spectra, that velocity must be attained in a very short time for the matter to reach 13 fm in 10 fm/c. Cascade calculations [11, 16], which do not include a softening to the equation of state, can fit the overall source size but under-predict the transverse dimension of the emitting source.

Without going into detail, the correlation function is principally determined by the particle separation distribution,
Here, $S_2$ provides the probability of emitting a particle of velocity $v$ at space-time coordinate $x_2$, and $g(v, \mathbf{r})$ gives the chance that two particles of the same velocity are separated by $\mathbf{r}$ on their asymptotic trajectories. Since the measured correlation function, $C(q, \mathbf{r})$, is itself a six-dimensional function, one can only extract three dimensions of space-time information about the source for any velocity $v$. Thus, time has to be extracted by assuming a symmetry between the spatial characteristics in the two transverse directions.

The dimension of the $g(v, \mathbf{r})$ in the outwards (parallel to $v$) direction is then stretched if the source is long-lived, i.e.,

$$R_{out}^2(v) = R_{out}^2 + v^2 \tau^2(v), \quad R_{side} \approx R_L$$

where $R_L$ is the transverse spatial size of the source. Thus, the comparison of $R_{out}$ to $R_{side}$ provides information regarding the length of time over which emission occurs. In addition to the outwards and sideways dimensions, the longitudinal dimension, $R_{long}$, also provides information about the temporal characteristics of the emission. The asymptotic probability cloud describing particles moving asymptotically with zero rapidity is confined to a fraction of the overall longitudinal extent of the source since high-rapidity sources are unlikely to emit particles with zero rapidity. If the distribution of sources are uniformly spread through all rapidity, and if the position of sources are described by a Bjorken (no longitudinal acceleration) expansion, $z = \nu_{source} t$, the size of the probability cloud is determined by the velocity gradient,

$$R_{long} = \frac{\nu_{therm}}{d\nu/dz} = \nu_{therm} \tau.$$  

Here, $\tau$ is the mean time at which breakup occurs. For transverse masses much larger than the temperature, $M >> M_T + p_T^2 >> T$, the thermal velocity of the longitudinal motion is determined by the simple form [3, 19, 21],

$$\nu_{therm} = \sqrt{T/M_T}.$$  

This is often referred to as $M_T$ scaling. After estimating the breakup temperatures from RHIC with other means, one can use the measurement of $R_{long}$ to determine the mean emission time. This well complements the comparison of $R_{out}$ to $R_{side}$, which provides insight into the variance of emission times.

Thus, a measure of $g(v, \mathbf{r})$ can be used to determine both spatial and temporal information about the source. The ability to extract $g(v, \mathbf{r})$ from correlation measurements hinges on the ability to invert the expression,

$$C(v, q) = \int d^3 r g(v, \mathbf{r}) |\psi(q, \mathbf{r})|^2,$$

where $\psi(q, \mathbf{r})$ is the relative wave function of two particles with asymptotic relative momentum $q$, and $v$ is the velocity of the two-particle center of mass, which will always be assumed to be zero by a fortuitous choice of reference frame. For identical spinless bosons, the relative wave function is particularly simple to invert, $|\psi(q, \mathbf{r})|^2 = 1 + \cos 2qr$. One can then perform a Fourier transform and retrieve $g(v, \mathbf{r})$. It has become standard procedure for experimentalists to divide $\pi \pi$ and $\pi \pi$ correlation functions by a Coulomb correction factor so that the resulting object can be treated as if the pions were never charged and $g$ can be found by a Fourier transform.

Once the inversion has been identified as a Fourier transform it is especially easy to relate $g(\mathbf{r})$ and $C(q)$ for Gaussian sources. Unfortunately, it is not so simple to relate $g(\mathbf{r})$ to $C(q)$ for pairs where the interaction is more complicated. Even the simple Coulomb interaction leads to an opaque form for the wave function to be inserted into Eq. (5). Figure 1 shows correlation functions for the $pK^+$ interaction calculated with Coulomb interactions assuming a Gaussian source of $R_{out} = 8\text{ fm, } R_{side} = R_{long} = 4\text{ fm}$. Results are also displayed for the corresponding classical expression [13] where,

$$C(q) = 1 + \exp \left\{-4q_{out} R_{out}^2 - 4q_{side} R_{side}^2 - 4q_{long} R_{long}^2 \right\}.$$
averages over all directions of \(q\), one can see that the classical description has a simple form at large \(q\),

\[
(8) \quad \frac{d^2\langle q\rangle}{d^2q} = \sqrt{1 - \frac{2n}{qr} - \frac{\eta}{qr}}
\]

where \(\eta = \mu q^2/\epsilon\) with \(\mu\) referring to the reduced mass. Thus, the correlation behaves as \((\mu q^2/\epsilon)<1/4\) for large \(q\).

The sensitivity of the correlation function to the shape can be seen by plotting the correlation as a function of the direction of \(q\) relative to the outwards axis. This is shown in Fig. 2. Again, the classical and quantum results agree well for larger relative momentum. At small relative momentum, the quantum calculation shows little sensitivity to the direction. At large relative momentum, the shape continues to scale as \(1/q^2\) which allows one to use large bins in \(q\).

However, inside the range of the strong interaction the wave functions are determined by phase shifts and integrals of the wave function over a range of \(q\). The origins of the particles are more likely to be in direction \(\cos\theta = \pm 1\), the particles traveling directly toward the deflecting charge will be directed away from the \(\cos\theta = \pm 1\) directions by the repulsive Coulomb force.

Strong interactions also distort wave functions and thus potentially provide the means to extract size and shape information. For distances beyond the scale of the strong interaction, the wave functions are determined by phase shifts \[\Delta\phi\] of the phase shifts \([22]\).

Thus, \(W\) can be expressed in terms of derivatives of the phase shifts and integrals of the wave function over a range in which the wave function is known. Since these integrals can be performed analytically, \(W\) can be expressed in closed form \([22]\).

Examples of the effects of the strong interaction are shown in Figs. 1 and 2 for \(pK^+\) correlations and for \(p\pi^+\) correlations in Fig. 3. The strong interaction has little effect on \(pK^+\) correlations since phase shifts vary little. Not surprisingly, the strong interaction provides a noticeable correlation for \(p\pi^+\) pairs with the relative momentum chosen to be near the peak of the \(\Delta\) resonance. The angular correlations for the \(R_{out} = 8\) fm, \(R_{side} = R_{long} = 4\) fm \(p\pi^+\) source shown in Fig. 3 are positive for \(\cos\theta\) near zero and become negative near \(\cos\theta = \pm 1\). The angular sensitivity has two sources.
The trajectories must pass directly over the origin. For cosθ, the interference will be strongest in the direction cosθ = ±1 rather than the minima which are seen in Fig. 3. The second source of angular dependence stems from the interference between the initial plane wave and the partial wave corrections. This interdependence stems from the interference between the scattered partial wave and the plane wave is strong in this direction. This diffractive behavior provides the leverage for extracting shape information from the correlation function.

Finally, I wish to make a connection of the analysis here to the Lednický analysis [7, 14, 15, 23, 27] which compares correlations for cosθ > 0 to those for cosθ < 0 for non-identical particles. The Lednický analysis is sensitive to the dipole moment of the distribution, i.e., the offset of one particle distribution to that of another type of particle with the same velocity. By analyzing the correlation as a function of cosθ, as is shown here, one can gain insight into the higher moments. For instance, the five quadrupole terms represented by $Y_{2m}$, along with the monopole moment represent the typical variables used to describe an elliptic anisotropy. Geometrically, these six variables correspond to the three axes of the ellipse along with the three Euler angles that describe the orientation of the ellipse. In terms of Gaussian parameters, these six variables would translate into the three Gaussian radii and the three cross terms [9]. These are precisely the terms that provide insight into the space-time nature of the source. In principal, much more information is available, and one could consider using these correlations to image the source and provide higher angular moments as well as a more precise radial (non-Gaussian) form [5, 20, 26]. Given the importance of HBT measurements from identical-pion and identical-kaon correlations, the experimental community should give the highest priority toward investigating the complementary information provided by non-identical correlations regarding source size, shape and lifetime.

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References

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