

# Correlation femtoscopy

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**Abstract** Recent results on particle momentum and spin correlations are discussed, particularly, in view of the role played by the effect of final state interaction. It is demonstrated that this effect allows for (i) correlation femtoscopy with unlike particles; (ii) study of the relative space-time asymmetries in the production of different particle species (e.g., relative time delays or spatial shifts due to collective flows); (iii) study of the particle strong interaction hardly accessible by other means (e.g., in  $\Lambda\Lambda$  system).

**Key words** particle • emission • correlations • spin • flow • delay • interaction

## Introduction

The momentum correlations of particles at small relative velocities are widely used to study space-time characteristics of the production processes, so serving as a correlation femtoscope. Particularly, for non-interacting identical particles, like photons or, to some extent, pions, these correlations result from the interference of the two-particle amplitudes due to the symmetrization requirement of quantum statistics (QS) [14, 20, 38].<sup>1</sup> The momentum QS correlations were first observed as an enhanced production of the pairs of identical pions with small opening angles (GGLP effect [14]). Later on, Kopylov and Podgoretsky [20, 38] settled the basics of correlation femtoscopy; they suggested to study the interference effect in terms of the correlation function, proposed the mixing techniques to construct the uncorrelated reference sample and clarified the role of the space-time characteristics of particle production in various physical situations.

The momentum correlations of particles emitted at nuclear distances are also influenced by the effect of final state interaction (FSI) [7, 8, 15, 19, 27, 28]. Though the FSI effect complicates the correlation analysis, it is an important source of information allowing for the coalescence femtoscopy (see, e.g., [34, 36, 43, 44]), the correlation femtoscopy with unlike particles [7, 8, 27, 28] including the access to the relative space-time asymmetries in particle production [30] and a study of particle strong interaction.

The two-particle correlation function  $R(p_1, p_2)$  is usually defined as a ratio of the measured two-particle distribution

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<sup>1</sup> There exists [20, 21, 38] a deep analogy of the momentum QS correlations of photons with the space-time correlations of the intensities of classical electromagnetic fields used in astronomy to measure the angular radii of stellar objects based on the superposition principle (HBT effect) [16]. This analogy is sometimes misunderstood and the momentum correlations are mixed up with the HBT correlations in spite of their orthogonal character and the absence of the classical analogy for correlations of identical fermions.

to the reference one obtained by mixing the particles from different events. It can be calculated [7, 8, 15, 19, 27, 28] as a square of the properly symmetrized stationary solution  $\psi_{\mathbf{k}^*}^{S(+)}(\mathbf{r}^*)$  of the scattering problem averaged over the relative distance vector  $\mathbf{r}^*$  of the emitters in the pair c.m.s. ( $\mathbf{k}^* = \mathbf{p}_1^* = -\mathbf{p}_2^* \equiv \mathbf{Q}/2$ ) and over the pair total spin  $S$ .

It is well known that the directional and velocity dependence of the correlation function can be used to determine both the duration of the emission process and the form of the emission region [20, 38], as well as – to reveal the details of the production dynamics (such as the freeze-out temperature and collective flows; see, e.g., [35, 39, 40] and recent reviews [10, 47]). It appears that with increasing energy of heavy ion collisions from AGS and SPS up to the highest energies at RHIC, the data on like pion correlations show a rather weak energy dependence [3] and point to the kinetic freeze-out temperature somewhat below the pion mass, a strong transverse flow (with the mean transverse flow velocity at RHIC exceeding half the velocity of light [48]), a short evolution time of 8–10 fm/c and a very short emission duration of about 2 fm/c. The short evolution and emission duration at RHIC are also supported by the correlation analysis with respect to the reaction plane [32]. The small time scales at RHIC were not expected in transport and hydrodynamic models [17, 45] and may indicate an explosive character of particle production (see, e.g., [11, 12]). This conclusion is supported by recent STAR measurement of the mean freeze-out phase space density  $\langle f \rangle_p$  at RHIC, indicating an increase with energy (a slight increase of the effective interference volume is not sufficient to balance the ~50% increase of  $dn/dy$  as compared with SPS) and centrality [41]. Extrapolation of the RHIC phase space density measurements to low transverse momenta predicts  $\langle f \rangle_p$  close to unity for central events, suggesting that significant multiboson effects (see [18, 31] and references therein) can be present at low  $p_t$  at RHIC. For the experimental status of boson coherence phenomena see [1, 2].

### Femtoscopy with unlike particles

The complicated dynamics of particle production, including resonance decays and particle rescatterings, leads to essentially non-Gaussian tail of the  $\mathbf{r}^*$ -distribution. Therefore, due to different  $\mathbf{r}^*$ -sensitivity of the QS, strong and Coulomb FSI effects, one has to be careful when analyzing the correlation functions in terms of simple models. Thus, the QS and strong FSI effects are influenced by the  $\mathbf{r}^*$ -tail mainly through the suppression parameter  $\lambda$ , while the shape of the Coulomb FSI is sensitive to the distances as large as the pair Bohr radius  $|a|$  (hundreds fm for the hadron pairs containing pions). These problems can be at least partially overcome with the help of transport code simulations accounting for the dynamical evolution of the emission process and providing the phase space information required to calculate the QS and FSI effects on the correlation function.

Thus, in a preliminary analysis of the NA49 correlation data from central Pb+Pb 158 AGeV collisions [22, 24, 25], the transport RQMD v.2.3 code was used. To account for a possible mismatch in  $\langle \mathbf{r} \rangle$  the correlation functions were calculated with the space-time coordinates of the emission

points scaled by 0.7, 0.8 and 1. The scale parameter was then fitted using the quadratic interpolation. The fits of the  $\pi^+\pi^-$ ,  $\pi^+p$  and  $\pi^-p$  correlation function indicate that RQMD overestimates the distances  $\mathbf{r}^*$  by 10–20%.

Recently, there appeared data on  $p\Lambda$  correlation functions from Au+Au experiment E985 at AGS [33]. As the Coulomb FSI is absent in this system, one avoids here the problem of its sensitivity to the  $\mathbf{r}^*$ -tail. Also, the absence of the Coulomb suppression of small relative momenta makes this system more sensitive to the radius parameters as compared with  $pp$  correlations [46]. The significant enhancement seen in the combined (4, 6 and 8 AGeV) correlation function at low  $k^*$  is consistent with the known singlet and triplet  $p\Lambda$  s-wave scattering lengths; the fit by the analytical expression (originally derived for  $pn$  system [27, 28]) yields the Gaussian radius  $r_0 = 4.5 \pm 0.7$  fm [24, 26], in agreement with the radii of 3–4 fm obtained from  $pp$  correlations in heavy ion collisions at GSI, AGS and SPS energies.

### Accessing particle strong interaction

In case of a poor knowledge of the two-particle strong interaction, which is the case for meson-meson, meson-hyperon or hyperon-hyperon systems, this can be improved with the help of correlation measurements.

In heavy ion collisions, the effective radius  $r_0$  of the emission region can be considered to be much larger than the range of the strong interaction potential. The FSI contribution to the correlation function is then independent of the actual potential form [13, 27, 28]. At small  $Q = 2k^*$ , it is determined by the s-wave scattering amplitudes  $f^S(k^*)$  at a given total spin  $S$  [27, 28]. In case of  $|f^S| > r_0$ , this contribution is of the order of  $|f^S/r_0|^2$  and dominates over the effect of QS. In the opposite case, the sensitivity of the correlation function to the scattering amplitude is determined by the linear term  $f^S/r_0$ .

The possibility of the correlation measurement of the scattering amplitudes has been demonstrated [24, 26] in a recent analysis of the NA49  $\pi^+\pi^-$  correlation data within the RQMD model. The fitted strong interaction scale, redefining the original s-wave scattering length of 0.23 fm, appeared to be significantly lower than unity:  $0.63 \pm 0.08$ . The recent BNL data on  $K_{l4}$  decays [37] also point to a similar shift (~20%). These results are in agreement with the two-loop calculation in the chiral perturbation theory with a standard value of the quark condensate [9].

Recently, also the singlet  $\Lambda\Lambda$  s-wave scattering length has been estimated [6, 24, 26] based on the fits of the NA49 data from Pb+Pb collisions at 158 AGeV. Though the fit results are not very restrictive, they likely exclude the possibility of a large singlet scattering length comparable to that of ~20 fm for the two-nucleon system.

The important information is also coming from  $\Lambda\Lambda$  correlations at LEP [5]. Here the effective radius  $r_0$  is substantially smaller than the range of the strong interaction potential, so the  $\Lambda\Lambda$  correlation function is sensitive to the potential form. In fact, the observed strong decrease of the correlation function at small  $Q$  can be considered as a direct evidence for the potential core [25]; particularly, the Nijmegen potential NSC97e yields a reasonable agreement with this data.

## Accessing relative space-time asymmetries

The correlation function of two non-identical particles, compared with the identical ones, contains a principally new piece of information on the relative space-time asymmetries in particle emission [30]. Since this information enters in the two-particle amplitude  $\psi_{-\mathbf{k}^*}^{(+)}(\mathbf{r}^*)$  through the terms odd in  $\mathbf{k}^*\mathbf{r}^* \equiv \mathbf{p}_1^*(\mathbf{r}_1^* - \mathbf{r}_2^*)$ , it can be accessed studying the correlation functions  $\mathfrak{R}_{+i}$  and  $\mathfrak{R}_{-i}$  with positive and negative projection  $k_i^*$  on a given direction  $i$  or, – the ratio  $\mathfrak{R}_{+i}/\mathfrak{R}_{-i}$ . For example,  $i$  can be the direction of the pair velocity, or any of the out ( $x$ ), side ( $y$ ), longitudinal ( $z$ ) directions. Note that in the longitudinally comoving system (LCMS), one has  $r_i^* = r_i$  except for  $r_x^* = \Delta x^* = \gamma_t(\Delta x - v_t \Delta t)$ , where  $\gamma_t$  and  $v_t$  are the pair LCMS Lorentz factor and velocity. One may see that the asymmetry in the out ( $x$ ) direction depends on both space and time asymmetries  $\langle \Delta x \rangle$  and  $\langle \Delta t \rangle$ . In case of a dominant Coulomb FSI, the intercept of the correlation function ratio is directly related with the asymmetry  $\langle r_i^* \rangle$  scaled by the Bohr radius  $a = (\mu z_1 z_2 e^2)^{-1}$ :  $\mathfrak{R}_{+i}/\mathfrak{R}_{-i} \approx 1 + 2\langle r_i^* \rangle/a$ .

A review of the simulation studies of the method sensitivity and the experimental results can be found elsewhere [22, 24, 26]. Here we discuss the out correlation asymmetries observed for  $\pi p$  and  $\pi K$  systems in heavy ion collisions at CERN SPS and BNL RHIC [22, 24, 26, 42]. These asymmetries are in agreement with practically charge independent meson production and a negative  $\langle \Delta x \rangle$  or positive  $c \langle \Delta t \rangle$  on the level of several fm (assuming  $m_1 < m_2$ ). The RHIC asymmetries seem to be somewhat overestimated by the RQMD model, while the NA49  $\pi^+\pi^-$  and  $\pi p$  asymmetries in central Pb+Pb collisions at 158 AGeV are in quantitative agreement with this model – it yields practically zero asymmetries for  $\pi^+\pi^-$  system, while for  $\pi^\pm p$  systems,  $\langle \Delta x \rangle \doteq -5.2$  fm,  $\langle \Delta t \rangle \doteq 2.9$  fm/c,  $\langle \Delta x^* \rangle \doteq -8.5$  fm. Besides, it predicts  $\langle x \rangle$  increasing with particle  $p_t$  or  $u_t = p_t/m$ , starting from zero due to kinematic reasons. The asymmetry arises because of a faster increase with  $u_t$  for heavier particle. In fact, the hierarchy  $\langle x_\pi \rangle < \langle x_K \rangle < \langle x_p \rangle$  is a signal of a universal transversal collective flow [22, 24, 26]; one should simply take into account that the mean thermal velocity is smaller for heavier particle and thus washes out the positive shift due to the flow to a lesser extent.

## Spin correlations

The information on the system size and the two-particle interaction can be achieved also with the help of spin correlation measurements using as a spin analyzer the asymmetric (weak) particle decay [4, 23, 25, 29]. Since this technique requires no construction of the uncorrelated reference sample, it can serve as an important consistency check of the standard correlation measurements. Particularly, for two  $\Lambda$ -particles decaying into the  $p\pi^-$  channel, the distribution of the cosine of the relative angle  $\theta$  between the directions of the decay protons in the respective  $\Lambda$  rest frames allows one to determine the triplet fraction  $\rho_t = \mathfrak{R}_t/\mathfrak{R}$  where  $\mathfrak{R}_t$  is the triplet part of the correlation function.

The spin correlations allow also for a relatively simple test of the quantum-mechanical coherence based on Bell-type inequalities derived from the assumption of the

factorizability of the two-particle density matrix, i.e. its reduction to a sum of the direct products of one-particle density matrices with the nonnegative coefficients [29]. Clearly, such a form of the density matrix corresponds to a classical probabilistic description and cannot account for the coherent quantum-mechanical effects, particularly, for the production of two  $\Lambda$ -particles in a singlet state. Thus, the suppression of the triplet  $\Lambda\Lambda$  fraction observed in multihadronic  $Z^0$  decays at LEP [5] indicates a violation of one of the Bell-type inequalities,  $\rho_t \geq 1/2$ .

## Conclusions

The particle momentum and also spin correlations give unique information on the space-time production characteristics including collective flows. Rather direct evidence for a strong transverse flow in heavy ion collisions at SPS and RHIC is coming from unlike particle correlation asymmetries. Being sensitive to relative time delays and collective flows, the correlation asymmetries can be especially useful to study the effects of the quark-gluon plasma phase transition. The correlations yield also a valuable information on the particle strong interaction hardly accessible by other means.

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## References

1. Achard P (and L3 Collaboration) (2002) Measurement of genuine three-particle Bose-Einstein correlations in hadronic  $Z$  decay. *Phys Lett B* 540:185–198
2. Adams J (and STAR Collaboration) (2003) Three pion HBT correlations in relativistic heavy ion collisions from the STAR experiment. *Phys Rev Lett* 91:262301
3. Adler C (and STAR Collaboration) (2001) Pion interferometry of  $\sqrt{s_{NN}} = 130$  GeV Au+Au collisions at RHIC. *Phys Rev Lett* 87:082301
4. Alexander G, Lipkin HJ (1995) Use of spin-correlations to study low energy  $\Lambda\Lambda$  and  $\bar{\Lambda}\bar{\Lambda}$  space symmetries and resonances. *Phys Lett B* 352:162–168
5. Barate R (and the ALEPH Collaboration) (2000) Fermi-Dirac correlations in  $\Lambda$  pairs in hadronic  $Z$  decays. *Phys Lett B* 475:395–406
6. Blume Ch (and the NA49 Collaboration) (2003) Results on correlations and fluctuations from NA49. *Nucl Phys A* 715:55c–64c
7. Boal DH, Gelbke CK, Jennings BK (1990) Intensity interferometry in subatomic physics. *Rev Mod Phys* 62:553–602
8. Boal DH, Shillcock JC (1986) Nuclear interferometry and thermal freeze out. *Phys Rev C* 33:549–556
9. Colangelo G, Gasser J, Leutwyler H (2000) The  $\pi\pi$  s-wave scattering lengths. *Phys Lett B* 488:261–268
10. Csörgő T (2002) Particle interferometry from 40 MeV to 40 TeV. *Heavy Ion Phys* 15:1–80
11. Csörgő T, Csernai LP (1994) Quark-gluon plasma freeze-out from a supercooled state? *Phys Lett B* 333:494–499
12. Dumitru A, Pisarski RD (2002) Explosive collisions at RHIC? *Nucl Phys A* 698:444–447
13. Gmitro M, Kvasil J, Lednický R, Lyuboshitz VL (1986) On the dependence of nucleon-nucleon correlations on the form of the short range potential. *Czech J Phys B* 36:1281–1287

14. Goldhaber G, Goldhaber S, Lee WY, Pais A (1960) Influence of Bose-Einstein statistics on the anti-proton proton annihilation process. *Phys Rev* 120:300–312
15. Gyulassy M, Kauffmann SK, Wilson LW (1979) Pion interferometry of nuclear collisions. I. Theory. *Phys Rev C* 20:2267–2292
16. Hanbury-Brown R, Twiss RQ (1956) A test of a new type of stellar interferometer on Sirius. *Nature* 178:1046–1048
17. Heinz U, Kolb P (2002) Early thermalization at RHIC. *Nucl Phys A* 702:269–280
18. Heinz U, Scott P, Zhang QH (2001) Multiboson effects in Bose-Einstein interferometry and the multiplicity distribution. *Annals Phys* 288:325–360
19. Koonin SE (1977) Proton pictures of high-energy nuclear collisions. *Phys Lett B* 70:43–47
20. Kopylov GI, Podgoretsky MI (1972) Correlations of identical particles emitted by highly excited nuclei. *Sov J Nucl Phys* 15:219–223
21. Kopylov GI, Podgoretsky MI (1975) Interference of two-particle states in elementary particle physics and in astronomy. *Sov Physics JETP* 42:211–218
22. Lednický R (1999) NA49 results on unlike particle correlations. NA49 Note number 210:1–46
23. Lednický R (1999) On correlation and spin composition techniques. *MPI-PhE/99-10:1–11*
24. Lednický R (2001) Femtoscopy with unlike particles. In: CIPPOG – Proc of the Int Workshop on the Physics of the Quark Gluon Plasma, 4–7 September 2001, Palaiseau, France, pp 1–6, (nucl-th/0112011)
25. Lednický R (2002) Lessons from momentum and spin  $\Lambda\Lambda$  correlations at LEP and SPS. In: Quark Matter 2002: XVI Int Conf on Ultrarelativistic Nucleus-Nucleus Collisions, 18–24 July 2002, Nantes, France. Posters, pp 1–14
26. Lednický R (2002) Progress in correlation femtoscopy. In: ISMD'02: Proc of the Int Symp on Multiparticle Dynamics, 7–13 September 2002, Alushta, Ukraine, pp 21–26, (nucl-th/0212089)
27. Lednický R, Lyuboshitz VL (1982) Final state interaction effect on pairing correlations between particles with small relative momenta. *Sov J Nucl Phys* 35:770–778
28. Lednický R, Lyuboshitz VL (1990) Final state interaction effect on correlations in narrow particle pairs. In: Proc CORINNE'90: Int Workshop on Particle Correlations and Interferometry in Nuclear Collisions, 28–30 June 1990, Nantes, France, pp 42–54
29. Lednický R, Lyuboshitz VL (2001) Spin correlations and consequences of quantum-mechanical coherence. *Phys Lett B* 508:146–154
30. Lednický R, Lyuboshitz VL, Erazmus B, Nouais D (1996) How to measure which sort of particles was emitted earlier and which later. *Phys Lett B* 373:30–34
31. Lednický R, Lyuboshitz VL, Mikhailov K, Sinyukov YuM, Stavinsky A, Erazmus B (2000) Multiboson effects in multiparticle production. *Phys Rev C* 61:034901
32. Lisa MA (for the STAR Collaboration) (2002) Azimuthally-sensitive pion HBT at RHIC. In: ISMD'02: Proc of the Int Symp on Multiparticle Dynamics, 7–13 September 2002, Alushta, Ukraine, pp 107–110, (nucl-ex/0301005)
33. Lisa MA (and E895 Collaboration) (2002) Laying the groundwork at the AGS: recent results from experiment E895. *Nucl Phys A* 698:185–192
34. Lyuboshitz VL (1988) Narrow pair correlations and bound state generation in the model of one particle sources. *Sov J Nucl Phys* 48:956–962
35. Makhlin AN, Sinyukov YuM (1988) Hydrodynamics of hadron matter under pion interferometric microscope. *Z Phys C* 39:69–76
36. Mrowczynski S (1992) On the neutron-proton correlations and deuteron production. *Phys Lett B* 277:43–48
37. Pislik S, Appel R, Atoyan GS et al. (2001) New measurement of  $K_{e4}$  decay and the s-wave  $\pi\pi$ -scattering length  $a_0$ . *Phys Rev Lett* 87:221801
38. Podgoretsky MI (1989) Interference correlations of identical pions. Theory. *Sov J Part Nucl* 20:266–282
39. Pratt S (1984) Pion interferometry for exploding sources. *Phys Rev Lett* 53:12191–221
40. Pratt S (1986) Pion interferometry of quark-gluon plasma. *Phys Rev D* 33:1314–1327
41. Ray RL (and STAR Collaboration) (2003) Correlations, fluctuations, and flow measurements from the STAR experiment. *Nucl Phys A* 715:45c–54c
42. Retière F (and STAR Collaboration) (2003) Non-identical particle correlation analysis as a probe of transverse flow. *Nucl Phys A* 715:591c–594c
43. Sato H, Yazaki K (1981) On the coalescence model for high energy nuclear reactions. *Phys Lett B* 98:153–157
44. Scheibl R, Heinz U (1999) Coalescence and flow in ultrarelativistic heavy ion collisions. *Phys Rev C* 59:1585–1602
45. Soff S, Bass SA, Dumitru A (2001) Pion interferometry at RHIC: probing a thermalized quark-gluon plasma? *Phys Rev Lett* 86:3981–3984
46. Wang F, Pratt S (1999) Lambda-proton correlations in relativistic heavy ion collisions. *Phys Rev Lett* 83:3138–3141
47. Wiedemann U, Heinz U (1999) Particle interferometry for relativistic heavy ion collisions. *Phys Rep* 319:145–230
48. Xu N, Kaneta M (2002) Hadron freeze-out conditions in high energy nuclear collisions. *Nucl Phys A* 698:306c–313c