Introduction

There is an attractive analogy for volume and spark ignition of nuclear fusion to be found in the well known types of internal combustion engines – i.e. the diesel and the gasoline engine. However, there is a large difference – the piston mass and that of the associated parts (equivalent in its function to that of the liner) corresponds to very large tamping of the exploded fuel, the heat losses during the expansion are small and therefore, the burn is high – approaching 100%. The burn in an inertially confined D-T plasma must be [2]:

\[ \frac{1}{\varepsilon} \geq \frac{10^8}{G} \]

where \( \varepsilon \) is the efficiency (in %) of generating ignited plasma out of a pool of thermal energy and \( G \) is the gain of the reactor (\( G = 1 \) corresponds to a zero output reactor).

In order to avoid a large circulating power, the gain \( G \) must be larger than 5. Unless completely new drivers are invented the \( \varepsilon \leq 5 \)(%) and \( \alpha = 1 \). This implies \( b > 30 \)(%), which as we will see, is difficult to achieve in cylindrical implosion.

If a spark could start a detonation in a tamped cylindrical D-T plasma (Fig. 1), then (similar to the gasoline engine) the energy necessary for the compression will be that required to obtain a propagation of a detonation in the plasma channel which is (Ref. [7]) using our definition of \( \kappa \):

\[ a' \rho' \geq 0.35 (\varphi' \kappa')^{-1} \]

where \( \varphi' = 1 \) for \( \alpha = 1 \div 2 \) (the equality in equ. (2) corresponds to a burn of about 1%).

Although this is more than twice as much as required for volume ignition, the plasma must not be heated to the ignition temperature \( T_{bg} \), a temperature \( T^* > T_{deg} \) will suffice.
The energy required for volume ignition is (Ref. [2], App. 1)

\[ W_{ig} = 4.5 \times 10^{13} \frac{\alpha}{\kappa \phi} \text{ (erg)} \]

and that for detonation (from equ. (2))

\[ W_{det} = 11 \times 10^{13} \frac{\alpha'}{\kappa' \phi'} \text{ (erg/cm)} \]

where \( \phi \) will not be much different from \( \phi' \) and \( \alpha' = T'/10^8 \).

Considering liner implosions of equivalent difficulty, we can assume that \( a/\kappa = a'/\kappa' \) and consequently

\[ W_{ig} \approx W_{det}' \]

**Ex.:** \( a = a' = 20 \text{ (mm)}, \kappa = \kappa' = 10 \), then \( n' = 1.6 \times 10^{13} \) and \( T_{deg} = 5 \times 10^7 \text{ (K)} \). Choosing \( T = 4 \times T_{deg} \) we have \( \alpha' = 2 \times 10^{-2} \) and \( q = 20 \).

Provided the spark-energy can be neglected with respect to the total \( W_{det} \), the spark ignition scheme is energetically much more advantageous than the volume ignition. The equation (1) can now be written as

\[ b = \frac{30aG}{q \epsilon} \text{ (%)} \]

A factor \( q = 10 \) makes here the difference between a feasible reactor and one demonstrating the principle of ICF only.

**Volume ignition**

Let us now discuss a typical case of volume ignition. The plasma has a line density \( N = 1.5 \times 10^{19} \text{ (ions/cm)} \), initial temperature \( T_0 = 200 \text{ (eV)} \) and radius \( a_0 = 1.2 \text{ (mm)} \). It is confined by a Z-pinch current \( I_0 = 1.41 \text{ (MA)} \). The initial plasma energy is 1.5 (kJ/cm). The liner speed is \( v_L = 2 \times 10^7 \text{ (cm/s)} \), its mass \( M_L = 20 \text{ (mg/cm – Ag plasma)} \), \( R = 2 \text{ (mm)} \), the \( W_k = 400 \text{ (kJ/cm)} \).

During the implosion the \( B_\phi \) flux (\( \phi \)) diffuses into the pinch and even more rapidly into the more resistive liner. At \( t_1 = 8.5 \text{ (ns)} \) the \( \phi \) between the pinch and the liner disappears almost completely and for \( t > t_1 \) the pinch touches the liner. However, the \( B_\phi \) at the pinch boundary remains high and therefore, the high temperature plasma is fairly well insulated from the liner during the compression period. (The requirement for the thermal insulation is \( B_\phi a >> 10^5 \). For \( \alpha \) confinement we need \( B_\phi a >> 2.7 \times 10^5 \text{ (Gauss xcm)} \).)

In Fig. 2 are plotted \( r_L \approx r_p \), \( \rho, T_e, T_i, W_\alpha \) and \( b \) over a period near the turnaround (9 ÷ 11.5 ns). The minimum \( r_L = 46 \text{ (mm)} \) is reached at 9.75 (ns), the \( \rho_m = 0.95 \text{ (g/cm}^3) \). There are small oscillations in \( \rho \), witness to the vibrations of the plasma column at a frequency \( f = v_L / a \).

The ignition starts where the curves of \( W_\alpha \) and \( b \) tend to zero, i.e. at \( t = 9.2 \text{ (ns)} \); the temperature at that moment is \( T = 7.5 \text{ (keV)} \). This is also the point where \( T_e \) separates from \( T_i \) because \( \alpha' \)’s are transferring their energy mainly to the electrons.

A clear sign that ignition occurred is that the maximum of \( (T_e + T_i) \) happens during the pinch expansion, the interval between \( \rho_{max} \) and \( T_{max} \) being 175 (ps). This is in agreement with the ignition criterion of Ref. [2], which requires that \( \rho a \geq 0.06/(\kappa \phi) \). In our case \( \kappa = (\tau_e / \tau) = 15 \) and \( \phi = 1.15 \) and therefore, our \( \rho a \) must be larger than 0.0035. In fact, at the maximum compression \( \rho a = 0.0044 \).

The burn reaches 5.4%, the \( W_\alpha = 2.2 \times 10^{12} \text{ (erg/cm)} \). Evidently for a more substantial burn one would require \( \rho a >> 0.06/(\kappa \phi) \). We have calculated cases with \( M_L = 30 \text{ (mg/cm)} \), others with a denser plasma and reached the conclusion that an energetically meaningful burn in volume ignitions can be reached only at the expense of larger \( v_L \) and larger plasma and liner energy – considerably larger than required for ignition.
Spark ignition in an inertially confined Z-pinch

Let us now address the problem of spark ignition and the detonation propagation. We have treated this subject in references [6] and [7], however, without referring to liner implosions, consequently the ignition and detonation simulations do not involve the tamping factor \( \kappa \).

Compressing a Z-pinch is particularly attractive since an \( m = 0 \) instability is able to produce rapid necking and a hot spot which, as has been shown in [7], can act as a spark of a detonation in the adjacent channel (Fig. 3). Of course the instability must grow from an initial axial perturbation (a kink, a void or a temperature gradient), which must be chosen so as to become significant at the moment of turnaround. One would hope that at the moment a large portion of the \( B_0 \) flux will be concentrated around the neck (Fig. 4), resulting in a current amplification. The beneficial action of the liner is, therefore, twofold:

a) it provides tamping (\( \kappa \)), lowering thus the \( (\alpha p) \) required for detonation;

b) it may help concentrating magnetic energy around the spark.

The numerical simulation of this process requires a complex 2D code [1]. Results are available for the compression of an isobaric pinch – the initial axial perturbation is present in the form of a \( T \) having a maximum at \( z = 0 \). The liner implosion causes a rarefaction in the central part of the pinch, which results in a hot spot and a spark. Let us describe a typical example.

The mass of liner \( M_L = 25 \) (mg/cm), its speed \( v_L = 3 \times 10^7 \) (cm/s), the plasma line – density at centre \( N = 5 \times 10^{19} \) (ions/cm), in the channel \( N' = 3 \times 10^{20} \) (ions/cm). The respective plasma temperatures are 120 (eV) resp. 20 (eV). The radii \( r_{0p} \) and \( a_0 \) (Fig. 1) are 2 and 1 (mm) resp. The initial pinch current \( I_0 = 2 \) (MA). In Fig. 5 is shown the evolution of the principal parameters at \( z = 0 \), near the moment of turnaround, as well as the fusion output \( W_{\alpha} \). In Fig. 6a are the snapshots of the \( \rho \) and \( T_1 \) distribution before, during and after the detonation for non-local \( \alpha \)-deposition. In Fig. 6b is the same scenario for local deposition.

From Fig. 5, we see that at \( z = 0 \) the product \( (\rho r_p') \) at maximum compression \( (t = 7.06 \) ns) is equal to 0.025. The criterion of volume ignition requires \( \rho r_p' > 0.06/(\kappa \rho) \). Our \( (\kappa \rho) = 20 \) and therefore, the central section of the pinch is well over the volume ignition requirement, also the \( T_1 \) at 7.06 (ns) is higher than the ignition temperature – hence the spark is strong enough to launch a detonation wave. It remains to check if our channel satisfies the detonation criterion (equ. (2)).

Using Fig. 6a we can plot \( r_p'(t) \) at \( z = 1 \) (mm) (Fig. 7). The \( (\rho r_p')_{\text{min}} = 63 \) (g/cm) and therefore, \( \rho' = 10 \) (g/cm) and \( (\rho r_p')_{\text{max}} = 0.063 \). From Fig. 7 we can estimate \( \kappa' = 3.5 \) and thus our \( \kappa' \rho = 0.22 \) much more than required by equ. (2).

The final \( W_{\alpha} = 3.4 \times 10^{12} \) (erg). As the total number of D-T couples is approx. \( 2.2 \times 10^9 \), the burn \( b = 2.8 \) (%). The \( d(W_{\alpha})/dt = \text{const.} \) part of the \( W_{\alpha}(t) \) curve is typical of a detonation wave in a cylindrical channel. From this we conclude that the detonation lasted about 140 (ps) and the speed of the detonation wave \( v_d = 6 \times 10^7 \) (cm/s).

The ignition and detonation process is depicted in Fig. 6a, where the non-local \( \alpha \)-deposition is represented using the \( \alpha \)-heat conductivity [6, 7]. As check on these results we have also simulated the same implosion assuming local \( \alpha \)-deposition (Fig. 6b). As expected, the detonation temperatures are higher and the detonation front has a character of a shock wave rather than a heat wave [6, 7].

The burn is 8.6%. The results of a simulation in which the \( \alpha \)-diffusion would be modelled perfectly will be somewhere between that of Figs. 6a and 6b.

Fig. 4. Formation of a pinch neck from an initial small \( m = 0 \) perturbation.

Fig. 5. Parameters of a spark ignition (non-local \( \alpha \)-deposition).
Conclusion

Spark ignition appears to be a more promising approach to ICF than volume ignition. The idea of spark ignition by a “hot spot” in a Z-pinch is not new [11], coming out of a long and varied experience with Z-pinch necking (plasma focus, fibre pinch, gas puff, etc.). The suggestion that a liner implosion and tamping could make the spark ignition more feasible came later [4, 5]. Recently the concept of spark ignition has been revived by laser and ion-beam specialists (fast ignitor). It seems, however, that an in-build $m = 0$ instability, is preferable to the injection of a picosecond beam (laser or ion) after a cold compression stage.

It seems that the logical way of using ignition in DT is that of amplifying the fusion energy in a conical channel to a level at which a detonation can be triggered in advanced fuels (e.g. pure D [7]). This leads to large energy bursts which, in order to be economic, should be of the order of several GJ/shot. The appropriate technology is “heavy” but accessible [10].

The technical means for the realization of liner implosion considered here should be sought along the following lines:

a) providing a fast, mm-sized, cylindrical liner by staging [8];

b) research in ablation driven cylindrical liners;

c) research in the induction of initial current $I_0$ by laser or electron beam injection.

Of course, all this would require expensive hardware, much more expensive than 2D numerical simulations!

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References