Introduction

It has been traditionally considered that a Z-pinch fusion reactor is based on the heating of a whole plasma column to high temperatures [1–3, 5]. However, an unrealistic energy input is needed for the plasma heating in this case. So, for example, in [1] these energies are estimated to be $10^{10}$ J, and a current of $10^9$ A, as mentioned in review [7]. The nuclear energy release is estimated to be 1 GJ [7]. Our approach [10] to the production of a positive energy yield consists of a small pinch zone heated to high temperature only. Experiments on Z-pinches always concern low and high temperatures and high density plasma regions that arise spontaneously in Z-pinch necks. A burn wave might be initiated in the Z-pinch column if in this small plasma region a Lawson-like condition is fulfilled. The nuclear energy, which is produced as a result of a burn wave spread, is controllable and is sufficient for the compensation of energy losses for the pinch production. The burn wave initiation energy is much lower than the energy necessary for heating the whole column. This study analyzes the conditions for starting the fusion burn wave propagation. A deuterium criterion for the applicability of Z-pinches to the fusion burn wave drive is also represented.

Condition for the fusion burn wave production in a Z-pinch

For production of the fusion burn wave in a D-T mixture within the Z-pinch one should heat the plasma at the neck, so that the energy released in charged reaction products ($\alpha$-particles) would be equal to or higher than the plasma heat content, i.e.:

\begin{equation}
Q_\alpha > Q_{\text{plasma}}
\end{equation}

Ion heating in the pinch should occur faster than its thermal spread in a radial direction. This condition is usually
satisfied at the necks since the rate of plasma compression towards the axis in the neck zone is determined by the Alfvén velocity, which is higher than the ion thermal velocity. Neck development simulation, taking into account the fusion energy release [11], confirm the need of condition (1) for the burn wave, but within a factor of about two.

To estimate the thermal plasma energy, $Q_{\text{plasma}}$, one can use the Bennett relation ($\pi r^2 n T = F/4c^2$). From this relation it follows that the thermal energy in the plasma column of length $h$, in the Z-pinch is equal to: $Q_{\text{plasma}} = 750 I^2 h$ ($Q_{\text{plasma}}$ is expressed in Joules; $h$ in cm; current, $I$, in MA).

If one takes into account the fact that $\varepsilon^* = 3.5$ MeV (energy of $\alpha$-particles) corresponds to each neutron from DT reaction, condition (1) can be represented in the form:

$\begin{align*}
(2) \quad Y_{\text{DT}} &> 750 h I^2 / (1.6 \times 10^{-19} s \times 3.5 \times 10^6) = 1.34 \times 10^{15} h I^2,
\end{align*}$

where $Y_{\text{DT}}$ is the neutron yield from D-T reaction ($Y_{\text{DT}} = Q_{\text{plasma}}/\varepsilon^*$); $h$ is the length of the zone radiating neutrons, in cm; $I$ is the current in MA; $1.6 \times 10^{-19}$ is a constant of conversion from eV to Joules. Expression (2) determines the neutron yield at which the ignition of the fusion burn wave start-up along the Z-pinch in the DT mixture is possible.

**Deuterium criterion for the fusion burn wave production**

Use of the DT-mixture is expected to initiate the fusion burn wave. However, in the majority of research laboratories there is no opportunity to use tritium in experiments. Nevertheless one can estimate the applicability of this mixture to initiate the fusion burn wave in this case, too. One can use a deuterium criterion for the estimation.

As it is known, in the absence of fusion burn wave, the neutron yield from a discharge with deuterium is about two orders of magnitude lower (precisely, 80 times lower) than the yield from plasma with the same parameters, using a DT-fuel (the ratio of neutron yields $Y_{\text{DD}}$ and $Y_{\text{DT}}$ is determined by the relation of sections D-D and D-T reactions only). Therefore, the condition for attaining of plasma parameters sufficient for initiation the fusion burn wave along the pinch in the D-T mixture (2) is the attainment of neutron yield from a similar deuterium plasma:

$\begin{align*}
(3) \quad Y_{\text{DD}} &> Y^* = 1.67 \times 10^{13} h I^2,
\end{align*}$

Let us name this condition as the D-criterion.

The expected dependence of $Y_{\text{DT}}/Y_{\text{DD}}$ on the parameter $Y_{\text{DD}}/Y^*$ for the Z-pinches is shown in Fig. 1. In the zone, where no fusion burn wave is produced ($Y_{\text{DT}}/Y^* < 1$), the ratio of neutron yields for the discharges with tritium and without is $Y_{\text{DT}}/Y_{\text{DD}} = 80$. At $Y_{\text{DD}}/Y^* \geq 1$ the burn wave is absent in deuterium plasma, meanwhile it emerges in the DT-mixture. Therefore, at $Y_{\text{DD}}/Y^* - 1$ the ratio $Y_{\text{DT}}/Y_{\text{DD}}$ should abruptly rise (Fig. 1). The shape of the curve for $Y_{\text{DD}}/Y^* > 0.5$ is obtained semiqualitatively.

A more precise dependence of $Y_{\text{DT}}/Y_{\text{DD}}$ on the parameter $Y_{\text{DD}}/Y^*$ can be produced from the results of a two-dimensional MHD numerical simulation of the neck development in the deuterium gas and DT mixture. The precomputations show that the shape of the curve in Fig. 1 depends on the following factors: initial plasma condition, radiation characteristics and turbulence of plasma, mechanism of propagation of the fusion burn wave. The fusion burn wave in a Z-pinch was already simulated by four scientific groups [4, 8, 9, 11]. The authors of these papers could define the given dependence that will serve to more exact and independent definition of possibilities of Z-pinches for the creation of controlled thermonuclear reaction.

**Discussion of deuterium criterion**

For a given D-criterion the values of $Y_{\text{DD}}$ and the current, in the system are, no doubt, experimentally determined, whereas the determination of $h$ is somewhat more complicated. In [9], the localization zone of neutron radiation from the plasma focus was determined. This zone mainly coincides with a zone of luminosity in the soft X-ray radiation. Therefore, one can use the luminosity zone size in the soft X-rays for estimating $h$. Table 1 includes the neutron yield, pinch current and the neutron radiation zone length for the Z-pinch facilities in Russia (plasma focus at the Kurchatov Institute and ANGARA-5).

Table 1. The neutron yield, pinch current, length of the neutron radiation zone and the evaluated $Y^*$ and $Y_{\text{DD}}/Y^*$ for plasma focus and ANGARA-5.

<table>
<thead>
<tr>
<th>Current of pinch (MA)</th>
<th>Length of area radiating neutrons (cm)</th>
<th>$Y^* = 1.67 \times 10^{13} h I^2$</th>
<th>Experimental neutron yield $Y_{\text{DD}}$</th>
<th>Value $Y_{\text{DD}}/Y^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF-I</td>
<td>1.5</td>
<td>2</td>
<td>7.5x10$^{13}$</td>
<td>10$^{11}$</td>
</tr>
<tr>
<td>Angara-5-1</td>
<td>2.0</td>
<td>1</td>
<td>6.7x10$^{13}$</td>
<td>10$^{12}$</td>
</tr>
</tbody>
</table>

As an example, a deuterium load on the 2 MA Angara-5 machine provides $Y_{\text{DD}} = 10^{12}$ from $h \sim 1$ cm [3]. From (3) $Y^* = 1.67 \times 10^{13} \times 1 \times 2^2 = 6.7 \times 10^{13}$. Angara-5’s yield is thus 67 times lower than required for burn wave initiation ($Y_{\text{DD}}/Y^* = 1/67$). If the neutron yield increases as the fourth power of the peak current ($Y_{\text{DD}} - I^4$) and $Y^* - I^2$, the cur-
rent must be increased \(67^{1/2} \sim 8 \) fold to get a burn wave in a DT load \( (Y_{DD}/Y^* = 1) \). This means that to get it, it is necessary to increase the current of ANGARA-5 up to 16 MA.

One should note that the origin of neutrons (from thermonuclear reactions or from acceleration) does not change criterion (1). The point is that the whole energy released in the Z-pinch plasma (independent of the energy source for ions in a pinch) is spent on the production of a fusion burn wave. Therefore, there is no necessity to perform a careful analysis of the neutron radiation drive mechanism in the Z-pinches. For initiating and propagating the fusion burn wave it is important that the neutron radiation originates from the pinch plasma and is not a result of the plasma irradiation by a deuterium beam from an external source.

**Analysis of plasma parameters at the neck necessary to initiate fusion burn wave in Z-pinch**

The fusion burn wave initiation along the pinch was simulated in many studies [6, 8, 10, 11] and one can obtain the plasma parameters for initiating the fusion burn wave from them. We used the results from [11], where the fusion burn wave was simulated simultaneously with the pinch neck development. In that study it has been found that it is necessary to produce the plasma with a temperature of 2 keV and \( \rho r > 0.25 \text{ g/cm}^2 \) (\( \rho r \) is the confinement parameter, i.e. the product of density and the neck radius) for the ignition and propagation of a fusion burn wave. The plasma outflow from the neck through the neck ends results in the plasma temperature rise from 2 to 10 keV. Then, the fusion burn wave, spreading from the neck along the pinch, emerges in the pinch.

Table 2 includes the data from [11] on the radius and density at the neck. With these parameters the fusion burn wave emerges there at currents 10, 30 and 100 MA. The DT fuel-mass, \( M^* \), which initiates the fusion, is shown in the fourth column of that Table. \( M^* = \pi r^2 h^*, \) where \( h^* \) is the pinch neck length. It is assumed, that \( h^* \approx 2r \). The minimal neutron yield from the D-T reactions and the minimal released energy in \( \alpha \)-particles to initiate the fusion burn wave are given in the fifth and sixth columns. These values are provided by the condition that the fusion burn wave is spread along the pinch for the distance, as a minimum, of the order of the neck length, \( h^* \), and releases the nuclear energy (in \( \alpha \)-particles) equal to the thermal plasma energy. From these data it follows that the DT-fuel mass, which initiates the fusion burn wave in the pinch, is low \( (10^{-5} - 10^{-2} \text{ g}) \). As a result, the energy needed for heating this plasma up to the fusion temperatures (100 Joule at the current of 10 MA) is not too high. The seventh column, Table 2, includes the source energy level necessary for the current production in the pinch (inductance is taken to be equal to 20 nH). From the Table one can see that a very small part of energy only \( (10^{-2} - 10^{-4}) \) is spent on the high-temperature plasma production for initiating the fusion burn wave. However, the low magnetic pinch energy efficiency of conversion into the energy of the igniting plasma is unavoidable, since the magnetic field is used for plasma compression up to high densities.

**Conclusion**

1. One can judge the chance to initiate the fusion burn wave in the Z-pinch with a D-T loading by a neutron yield from the Z-pinch height unit in the D-D reaction, using criterion (3). Criterion (3) can be used also for estimating how close the actual Z-pinch is to the achievement of the ignition conditions for the fusion burn wave.

2. According to criterion (3), the achievement of a high neutron yield from the unit of the pinch height, i.e. \( Y_{DD}/h \) is the most important. Therefore, the Z-pinch experiments in which one neck is produced [2] are very useful.

3. The fusion burn wave is principally possible at the pinch current of 10–30 MA. In this case, the minimal fusion heat release, at which this wave can be registered, is \( 10^5–10^6 \) Joules. Therefore our studies on the fusion burn wave production in the Z-pinch do not expect the energy release \( 10^9 \) J in the fusion radiation, as shown in review [7]. Burn wave ignition is possible at a nuclear release lower by 5–7 orders of magnitude.

4. For achievement of criterion (3) in pinches with deuterium it is sufficient to produce a neutron yield from one neck greater than \( 3 \times 10^{12} \) at a current of 10 MA or \( 3 \times 10^{14} \) at a current of 30 of MA.

As a result of the performed analysis for the fusion burn wave ignition in pinches the following programme of studies is proposed:

1. to concentrate efforts on the production of the highest ratio \( Y_{DD}/Y^* \), where \( Y^* = 1.67 \times 10^{13} \) hI\(^2\) in the Z-pinch.

2. to realize experiments in the D-T mixture in those Z-pinches for which \( Y_{DD}/Y^* \sim 1 \), i.e. the fusion burn wave will be ignited.

**Acknowledgment** The work on the search of experimental criteria for the Z-pinch ignition was initiated and supported by Smirnov V.P. The study was also supported by the Russian Fund of Fundamental Studies, Project No. 99-02-16658.
References

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