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The need of using anomalous resistivity due to Lower Hybrid Instabilities in plasma-magnetic field interfaces*

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Abstract Anomalous plasma can arise in dense plasmas as a consequence of the development resistivity of the Lower Hybrid drift Instability (LHI), which is triggered whenever the electron drift velocity becomes comparable to or larger than the thermal ion velocity. This effect was carefully evaluated for theta-pinch plasmas and later on used in the modelling of denser pinches, like Plasma Focus columns or Z-pinches, because of the diverging electron drift velocity expected in the classical equilibrium (uniform current, parabolic plasma density) pinch. Conceptually, the possibility of diverging drift velocities is far more a general situation than Z-pinches, and in this work we study it, in general, for plasma-magnetic field interfaces existing in other configurations, like that found in devices producing travelling current sheets (Plasma Focus devices, imploding pinches, etc.). We show that it is essential to account for this effect in steady state situations, and that it could be also important in the time varying ones.

Key words anomalous resistivity • dense magnetised plasmas • plasma instability

Introduction

Anomalous plasma resistivity can arise in dense plasmas as a consequence of the development of the Lower Hybrid Instability (LHI), which is triggered whenever the electron drift velocity, v_d, becomes comparable to or greater than the thermal ion velocity, v_i. Other instabilities can develop in plasmas (like ion acoustic, Buneman, etc.) but most of them require electron temperatures larger than ion temperatures (an unlikely situation in shock plasmas) and in any case they are also triggered by equivalent (or more stringent) conditions on v_d. The LHI effect was discussed and carefully evaluated for theta-pinch plasmas [4], and later on used in the modelling of the pinch stage in a Plasma Focus device [7] and in solid fibre Z-pinch plasmas [3, 5, 6]. The feasibility of using the theory developed in the context of theta-pinches to Z-pinch plasmas was particularly discussed in the works by Chittenden [2, 3].

The rationale behind the inclusion of this effect in Z-pinches, as mentioned by Robson when studying the possibility of obtaining radiative collapses [5, 6] is the fact that, the frequently used steady state profiles of current density, j (uniform) and plasma density, n (parabolic) for these pinches have a diverging v_d on the pinch border. This is quite true, but it is worthwhile to extend the analysis to more general situations. Conceptually, the possibility of diverging v_d seems to be far more a general problem than equilibrium Z-pinches configurations, and we will study this problem for plasma-magnetic field interfaces existing in other configurations, showing that it is important to account for this effect.

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General plasma magnetic field interfaces

The cylindrical steady state pinch studied by Robson is just an example of the plasma magnetic field interfaces that are likely to present the problem of diverging electron drift velocities. We will analyse the possibility of having this kind of problems in plasma-current structures with j and B perpendicular to each other, and varying in the direction perpendicular to them both (which will be denoted as y). The plasma extends in the y direction, with the boundary condition $j = 0$ and $n = 0$ both at the same (finite) y value, which will be denoted by a, and remain zero from there on. Such a configuration is quite general, and includes besides cylindrical pinches, cylindrical converging plasma sheets, plane plasma sheets and plane coaxial plasma sheets.

To fulfil the boundary condition for the current and particle densities, we have:

(1) $$j(y,t) = (y-a)^m f(y,t),\ y \le a;\ y > a$$

Where m is the positive constant and $f(y,t)$ is the regular function satisfying $f(a,t) \neq 0$ and a similar relationship holds for n.

Let us start with steady state situations. In these cases, the momentum balance equation can be written down as:

(2) $$k(ZT_e+T_i)\frac{\partial n}{\partial y}+n\frac{\partial k(ZT_e+T_i)}{y}=-jB$$

where Z is the ionisation level of the ions, and spatially varying and different electron (T_e) and ion (T_i) temperatures are allowed. Note that this does not imply the "classical" parabolic plasma density profile.

As both j and n go to zero at $y = a$, the problem is to evaluate the limit of j/n on the boundary. Using the L'Hopital rule together with the momentum balance equation, we find:

(3) $$\lim_{y\to a}\frac{j}{n}=\lim_{y\to a}\frac{\partial j/\partial y}{\partial n/\partial y}\infty\lim_{y\to a}\frac{\partial j/\partial y}{(jB)}\infty\lim_{y\to a}\frac{1}{y-a}$$

Here use have been made of equation (2) to evaluate that $\partial n/\partial y \to -jB$ when $y \to a$ (assuming that the gradient of temperatures is bounded on the border, which is quite reasonable), and also of equation (1) for the evaluation of $\partial n/\partial y$. Hence, the limit is divergent. This means that in this type of plasma-magnetic field configurations, it is essential to include LHI driven anomalous effects to obtain the plasma current structure.

Let us now consider this situation in a more general case of time dependent plasma interfaces evolution. The momentum equation writes:

(4) $$Mna_y+k(ZT_e+T_i)\frac{\partial n}{\partial y}+n\frac{\partial k(ZT_e+T_i)}{\partial y}+(\nabla.\Pi)_y=-jB$$

where a_y is the plasma acceleration, M the ion mass, Π is the viscous stress tensor and $(\nabla.\Pi)_y$ is the y component of its divergence. Neglecting the viscous term in equation (4) by the time being, now the limit becomes:

(5) $$\lim_{y\to a}\frac{j}{n}=\lim_{y\to a}\frac{\partial j/\partial y}{\partial n/\partial y}\infty\lim_{y\to a}\frac{\partial j/\partial y}{(jB+n(Ma_y)+k\partial(ZT_e+T_i)/\partial y)}$$

An inspection to the definition of j shows that, if $m \le 1$, the limit is divergent because, besides of the zero in the denominator, $\partial n/\partial y$ diverges at $y = a$. In the case of $m > 1$, the limit is still uncertain. However, repeating this procedure as many times as needed (that is, deriving the numerator and denominator as many times as the integer part of m), the numerator will diverge (or tend to a constant), while the denominator (containing n and lower order derivatives of j) will keep tending to zero, provided that the derivatives of f, a_y, T_e, T_i and B are all bounded (which should be, on physical grounds). Hence, under such boundary conditions, the drift electron velocity is always divergent on the border, and LHI effects should be included in the description of plasma behaviour.

The inclusion of viscous forces changes the picture. In fact, these forces contain functions of the plasma velocity and its derivatives, multiplied by the ionic viscous coefficients ξ_o (independent of n) and ξ_1 (which includes $\omega_{ci}\tau_i$ effects and can depend on n) or its first derivatives. Therefore, in contrast with the inertial and temperature gradient terms, several of the viscous ones do not contain n as a factor. As a consequence, on the border, the density gradient does not tend to zero anymore, but tend to the value of the surviving viscous terms. Hence, and except the case $m < 1$, j/n tends now to zero on the boundary.

In time dependent problems, it is difficult to predict how j will tend to zero on the border, which is determined by the rate of change of the total current flowing into the plasma and the magnetic diffusion equation. For this reason, we cannot make definite statements concerning the behaviour of j/n in this case. Furthermore, we must note that even if divergences are removed on the boundary, this does not preclude that this quotient in other positions reaches values sufficiently large to trigger LHI, generating an anomalous resistivity. Such event will modify the magnetic field diffusion and hence, j profiles everywhere. Hence, LHI effects should be included, for safety reasons, in any modelling of the evolution of this kind of plasmas.

Final remarks and conclusions

We have shown that, in any assumed sharp plasma-magnetic field interfaces, defined by the request that both j and n go simultaneously to zero at a co-ordinate, the electron drift velocity diverges at this co-ordinate in steady state situations. In time dependent situations, only viscous effects could (but not necessarily) remove the divergence. This unphysical situation has not been noticed until recently [2, 5] probably because v_d is not physical variable for a directly looked.

The arguments presented in this work hold for "straight" plasma-magnetic field interfaces, without curvature in the j direction. The analysis of curved structures is out of the scope of this work, but it is difficult to imagine that its inclusion would yield a different result.

Due to the fact that large values of v_d will trigger LHI, increasing the plasma resistivity, it is then advisable to use such an effect in any modelling of this type of plasmas. When this is done [2, 5, 6], plasma and current densities cease to be "sharp", and become spread, so that no precise border of the plasma structure exists.

In 1-D, time dependent MHD codes, divergences of v_d may not actually be observed because the discretization procedure removes them. We are currently studying one such a Lagrangian code (essentially similar to the 2-D version described in [1]), with and without anomalous resistivity. We have observed in many situations that v_d/v_i on the border increases without limit when increasing the number of cells, while all other parameters converge to reasonable values. Also, we have found that the values of this quotient become easily much larger than 1, in further support of the need of including always LHI effects.

One key point in our line of reasoning is the requirement that $n = 0$ on the border of the structure. It could be argued that, in essentially all of the experimental configurations, a residual low density plasma might exist in the "vacuum" magnetic field region, thus removing the divergence (or, more precisely, keeping v_d sufficiently smaller than v_i). The neutral gas surrounding fibres, gas puffs and gas embedded pinches can probably be partially ionised through photo-ionisation and electron thermal conduction from the dense plasma, and sweeping magnetic pistons might "leak" some low density plasma behind them. However, even if such were the case, this low-density plasma should also conduct a small fraction of the current, leading in any case to spread distributions. From the point of view of modelling, if one includes a low-density plasma region surrounding the plasma core, one must consistently move the vacuum magnetic field border condition to the end of this low-density region.

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